Chapter 19

Setting the Cost of International Capital

This chapter deals with how to set the cost of capital, which is the discount rate used in Capital Budgeting. The chapter title adds one word: international. Note that what is said to be international is *capital*. The title does not say "the international cost of capital", as the 1994 book (and many others) did: such a title would have suggested that there is something like a national cost (for domestic projects, presumably) and, next to that, an international cost, for transborder investments. No, there is just one cost of capital, and that capital is international. Shares of large corporations are held by people everywhere; and, equally important, even shareholders of smaller, more locally held firms still invest part of their wealth in foreign stocks. This has two implications for the way the cost of capital is to be set. First, managers should ask the question how much risk this project adds to an internationally diversified portfolio instead of to a local market portfolio (the traditional method), and set a cost of capital that is commensurate with this international risk. Second, management has to take into account that the expected return differs depending on what currency the shareholder uses as the (quasi-)real numeraire; and so does the risk-free rate that serves as one benchmark item in the model. That is, when setting the cost of capital, the issue of exchange risk has to be taken into account too, in the sense of investors having different numeraires in which they are doing, or supposed to be doing, their optimum-portfolio calculations.

There is a second—and largely independent—issue related to exchange rates: how do we bring expected cash flows and cost of capital in line with each other. The issue arises because when, say, an Australian firm invests in India, the expected future cash flows are normally first expressed in Rupees. Yet, the argument typically goes, the Australian owners care about Australian Dollars only—we'll make this argument more precise as we proceed—and the cost of capital we would estimate is probably expressed in AUD. One cannot discount INR cash flows using an AUD discount rate. So at one point we need to go from INR to AUD.

There seems to be two ways we could go about this, similar to what we did earlier for risk-free cash flows. As we saw, a risk-free claim on INR 1 can be PV'ed in INR terms first, by discounting the INR cash flow (unity) at the INR risk-free rate and this value is then translated into AUD at the going spot rate. Alternatively, we can translate the future cash flow into AUD using the expected future spot rate, and then discount at an AUD rate that takes into account the risk. Both are linked via the forward rate as the risk-adjusted expectation and CIP:

$$\begin{aligned} \frac{E_t(\tilde{S}_T)}{1+r_{t,T}+RP_{t,T}} &= \frac{F_{t,T}}{1+r_{t,T}}, && \text{(F=CEQ)} \\ &= \frac{1}{1+r^*_{t,T}} S_t. && \text{(CIP)} \qquad (19.1) \end{aligned}$$

Similarly then, in case of a risky FC cash flow, we could first translate the *future* INR cash flows into AUD using the expected future spot rate, and then PV these using an AUD discount rate, set *e.g.* on the basis of the standard Capital Asset Pricing Model (CAPM), the way Australians would value a domestic Australian project. Alternatively, we could argue that the Australian ownership hardly matters, and simply conduct the entire cost-benefit analysis in INR, the way an Indian owner would do: take INR cash flows, and discount at the rupee rate of return. Having found the value in INR, we then translate the *present* value into AUD. And if that second solution really works, exchange-rate forecasts and currency risk can be totally eliminated from the analysis, it would seem.

In this chapter we show that the above analysis is quite incomplete. The main lessons to be remembered from this chapter are the following:

- **Translation of FC cash flows requires more than just an expected exchange rate** Suppose we follow the first route and translate our Rupee cash flows, $\tilde{C}^*_T$, into AUD. What we need are expected AUD cashflows; but the expectation of a product, $\mathrm{E}(\tilde{C}^*_T \tilde{S}_T)$ involves not just the expectations of $\tilde{C}^*_T$ and $\tilde{S}_T$, but also the covariance between the two.

  This, at first sight, makes the first route even more difficult. All the more reason to go for the alternative one, then? Unfortunately, this alternative would not always work:

- **Host-currency v home-currency valuation** Valuation in Rupees, the way an Indian investor would do it—using the Rupee risk-free rate and a premium for market risk measured in Rupees—should produce the same result, after translation, as valuation *à l'Australienne* only if the Indian and Australian capital markets are well integrated. Indeed, if investors from each country can freely invest in each other's market (and possibly in other markets), arbitrage flows would occur if the value to Australians were different from the value to Indian investors (after translation into a common currency).

  In the case of India integration of the capital market into the mainstream work market is doubtful, for the time being. But even if it were true, the Indian-

Rupee approach would still not exonerate us from thinking about expected exchange rate changes and exchange covariance risk:

- **In open markets, exchange risk affects any cost of capital ...** In principle, exchange risk enters asset pricing as soon as the investor base for which we want to value the project is part of an international market. Thus, Australia being part of a nearly worldwide capital market, an International CAPM (i-CAPM) should be used whether the project is situated at home or abroad. Intuitively, in an international capital market, asset prices result from the interaction of portfolio decisions by people from many different countries, each having their own currency. Exchange risk makes people disagree about expected returns and risk; for example, the AUD treasury bill is risk-free to Australians, but not to Canadians or Japanese. This heterogeneity of perspectives does affect asset pricing, and introduces currency risk premia into the CAPM, in principle one for each currency area that is part of the international capital market.

  Thus, in a way things are even more complicated than your worst fears might have been: you need expected returns on *all* currencies in the international capital market, and covariances for your project with each of these currencies. In addition, exposures to exchange rates are even harder to estimate than betas. Fortunately, ...

- **... but currency risk premia are small** The literature on the forward rate as a predictor of futures spot rates shows that, while the currency risk premium is surely not a constant, it is small and seems to fluctuate around zero. So one could use a shortcut, omitting the forex items in the i-CAPM formula, so that it looks rather like the familiar domestic CAPM. Two differences remain: the market portfolio is the world-market index rather than a domestic one, and the market beta is from a multiple regression with all exchange rates included.

The discussion can be summed up as follows.

(*i*) **Which CAPM** You use a (possible simplified) i-CAPM when the *home* country is part of an international capital market; the domestic model works only for segmented home markets.

Note, incidentally, that this holds for any investment, whether at home or abroad.

(*ii*) **Which currency** If home and host are *both* part of the same international market, either currency will do for valuation purposes; otherwise only the investors' HC can be used.

This chapter addresses these issues in the following order. First we discuss the effect of capital-market integration or segmentation on the capital-budgeting procedure (Section 19.1), notably which should come first, translation from FC to HC or discounting. The bulk of the chapter then relates to the determination of the cost of capital. In the second section, we present the traditional single-country CAPM, starting from the efficient-portfolio problem familiar from basic finance courses. In Section 19.3, we explain how to modify this model when assets are priced in an international market. The case that we discuss is one where capital markets are integrated across many countries, but where imperfections in the goods markets create real exchange risk. Section 19.4 concludes with a review of the implications of this chapter for capital budgeting.

19.1 The Link between Capital-market Segmentation and the Sequencing of Discounting and Translation

To initiate our discussion of the effect of capital market integration or segmentation on the capital budgeting procedure, we explain why capital budgeting can be done in terms of foreign currency when the home- and host-country capital markets are integrated, and how the procedure is to be modified when the home- and host-country capital markets are segmented from each other.

Almost inevitably, capital budgeting starts with cash-flow projections expressed in host (foreign) currency. When one prepares cash flow forecasts there is no real choice but to start from currently prevailing prices for similar products in foreign currency. On the basis of this you set your own price(s), taking into account the positioning of the product(s). Then you try to figure out production costs on the basis of data from similar plants and local wages and other input costs. (Don't forget the initial inefficiencies, the learning curve. And think of possible price drops later when competition catches up or the rich segment has been creamed off or excitement about your product wanes.) This way you obtain cash-flow forecasts, all typically at current (i.e. constant) FC prices. Finally you adjust the figures for expected foreign inflation. This practice stems from the empirical fact, noted in Chapter 3, that prices in any given country are sticky (apart from general inflation) and to a large extent independent of exchange rate changes.

DoItYourself problem 19.1

You could think of an alternative version of the final step: translate the constant-prices cash flow into HC and then adjust for inflation in the investor's home country. Show that this unattractively assumes relative PPP, at least as an expectation. Assume risk-free cash flows at constant FC prices, for simplicity.

So we usually end op with expected cash flows in FC. However, the ultimate purpose of capital budgeting is to find out whether the project is valuable to the parent company's shareholders. The correct procedure is to see how *they* price similar existing projects. We can see that only by looking at their own capital market; that is, we use the shareholders' home capital market to get the risk-free rate and the estimated risk premium. But this delivers a cost of capital in HC units, which can only be used to discount HC expected future cash flows. For example, one would not use a low JPY-based discount rate to PV a stream of Zimbabwe Dollar cash flows. In short, although the natural input data are cash flow forecasts expressed in foreign currency, in principle we have to make the translation from foreign currency to home currency before we can discount. To what extent would it be acceptable, instead, to discount FC cash flows at a FC rate, and then to translate the FCPV into HC using just the current spot rate? After all, this is the way a local investor goes about the valuation.

This type of valuation in foreign currency, as if the owner were a host-country investor, is correct if the host- and home-country financial markets are integrated, that is, if there are no restrictions on cross-border portfolio investment between the two countries and if investors effectively hold many foreign assets. Indeed. the implication of market integration is that all investors, regardless of their place of residence, use the same cost of capital when they compute the price of any given asset (in some given common currency) from the expected cash flows of this asset (expressed in the same common currency). One way to explain this claim is by contradiction. If investors from countries A and B used a different cost of capital when computing the price of some given asset (in some given base currency) from the asset's expected cash flows (measured in the same base currency), then the price of the asset in country A would differ from the price of the same asset in country B. The resulting arbitrage opportunities would lead to international trading in the shares until the price difference disappeared. By equating prices across countries, international arbitrage also equates the costs of capital that various investors use when linking the asset's price to the expected cash flows paid out by the asset.[1] Thus, in integrated markets, a home-country investor and a host-country investor fully agree about the project's value.

In the perfect-markets approach of Chapter 4, perfected integration was taken for granted. But in the case of FDI into emerging countries it is not always obvious that integration is a reasonable approximation, even though restrictions are gradually being abolished in many countries. The problem is that in segmented markets one cannot simply value a foreign cash flow as if it were owned by host-country investors. In the absence of free capital movements, there is no mechanism that equates prices and discount rates across the two markets. Thus, to the managers of the parent firm, the relevant question becomes: What price would home-country investors normally be prepared to pay for the project? As we saw, the way to proceed is to identify cash flow patterns that have similar risks and that are already priced in the home-country capital market. Once we have identified a similar asset that is already priced in the home capital market, we can then use the same discount rate for the project that

[1]Investors that are not willing to pay a high price then sell to others that are. Portfolio rebalancing also modifies the risk: the risk of holding Samsung shares is very different depending whether this company represents 90 % of one's portfolio versus just 0.1% of a well-diversified package of securities. So reducing the weight of one asset, and replacing it by others that offer more diversification, lowers required returns for that asset and increases the price one is willing to pay for it. In the end, when both domestic and foreign investors hold very similar portfolios, required returns would converge.

we want to value as that for the traded assets. To implement this procedure, we need a theory, like the Capital Asset Pricing Model, to tell us what types of risk are relevant, how these risks should be measured, and what return is expected in the home-country capital market in light of the project's risks. Since we use the home-country capital market as the yardstick, the discount rate is the required return in home currency—and if the cost of capital is expressed in home currency, we have to translate the expected cash flows and their risks from foreign currency into home currency before we discount.

For such a translation, we need expected values for the future spot rates for various maturities. In fact, we need also the covariance. If $\tilde{C}^*$ denotes the cash flow in FC and $\tilde{C}^*\tilde{S}$ the cash flow in HC, then

$$E(\tilde{C}^*\tilde{S}) = E(\tilde{C}^*) \times E(\tilde{S}) + \text{cov}(\tilde{C}^*, \tilde{S}). \tag{19.2}$$

You may have noticed the covariance effect in the Freedonian Crown exposure example in Chapter 13, which we reproduce here:

Example 19.1

A British company is considering a project in Freedonia. Assume that the Freedonian crown (FDK) cash flow can take on either of two equally probable values, FDK 150 or FDK 100, depending on whether the Freedonian economy is booming or in a funk. Let there also be two, equally probable time-T spot rates, GBP/FDK 1.2 and 0.8. Thus, measured in GBP, there are four possible cash flows: 150 × 1.2 = GBP 180, 150 × 0.8 = GBP 120, 100 × 1.2 = GBP 120, and 100 × 0.8 = GBP 80. These numbers are shown in Table 19.1. In each cell, we also show the joint probability of each particular combination. When the FDK is expensive, a recession is more probable than a boom because an expensive currency means that Freedonia is not very competitive. The inverse happens when the crown is trading at a low level; then it is more likely that the economy will be booming. These effects are reflected in the probabilities shown in each of the four cells in Table 19.1.

Table 19.1: **Cash flows for the Freedonian project**

	State of the economy			
	Boom: C^*=150	Slump: C^*=100	Prob(S)	E($\tilde{C}\|S$)
S_T=1.2	p = 0.15; C=180	p=0.35; C=120	0.50	138
S_T=0.8	p = 0.35; C=120	p= 0.15; C= 80	0.50	108
Prob(C^*)	p = 0.50	p = 0.50		

The expectations of the exchange rate and the FDK cash flows are easily calculated as

$$E(\tilde{S}) = (0.50 \times 1.2) + (0.50 \times 0.8) = 1.00, \tag{19.3}$$
$$E(\tilde{C}^*) = (0.50 \times 150) + (0.50 \times 100) = 125. \tag{19.4}$$

But the expected cash flow is not 1.00 × 125 = 125:

$$E(\tilde{S}\tilde{C}^*) = (0.15 \times 180) + (0.35 \times 120) + (0.35 \times 120) + (0.15 \times 80) = 123. \tag{19.5}$$

The shortfall of 2 (=125 − 123) is due to the fact that high FDK cash flows tend to go together with low exchange rates and vice versa. This effect is lost if one just multiplies through the two expectations, because that computation implicitly assigns probabilities 0.25 to each cell:

$$\begin{aligned} E(\tilde{S})E(\tilde{C}^*) &= [(0.50 \times 1.2) + (0.50 \times 0.8)] \times [(0.50 \times 150) + (0.50 \times 100)], \\ &= (0.25 \times 180) + (0.25 \times 120) + (0.25 \times 120) + (0.25 \times 80). \end{aligned} \tag{19.6}$$

So when we use the Translate First approach, the expected GBP cash flow is GBP 123 not 125.[2] This number is to be discounted at the appropriate home currency discount rate, that is, the GBP risk-free rate plus a risk premium that reflects the risk of the GBP cash flows to the British investor.[3] The Capital Asset Pricing Model, to be discussed in Sections 19.2 and 19.3, provides a way to estimate the appropriate discount rate.

While the Translate First approach is very general, it requires explicit exchange-rate forecasts, and the covariance. These do not come in explicitly if we take the Discount First route, and compute a PV for the expected flow $E(\tilde{C}^*) = 125$, using the FDK risk free rate and risk premium. This would be all right if the Freedonian and British markets are well integrated.

We have seen how to obtain expected cash flows, but not how to obtain appropriate discount rates when cash flows are risky. This is the task in the remainder of this chapter. Section 2 reviews the single-country CAPM. Section 3 extends the model to a multi-country setting.

19.2 The Single-Country CAPM

Our discussion of the traditional (single-country) CAPM starts from asset demand theory. The key assumption of this asset demand theory is that investors rank portfolios on the basis of two numbers, the expected nominal portfolio return and the variance of the nominal portfolio return. Implicit in the use of nominal returns is an assumption that inflation is deterministic, or at least that inflation uncertainty has little impact on asset pricing. The theory of optimal portfolios, as developed by Markowitz (1952), can also be interpreted as a theory that tells us how expected

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[2] In the above example, the cov-correction is relatively small. But the link between exchange rate and cash-flow is weak too, in the above story: it just works via general economic activity. In reality, there often is a strong, direct link, for instance if the firm is an exporter or importer, and then the covariance would be bigger.

[3] Recall that if capital markets within, say, the OECD are well integrated, the UK value would also be correct for any other investor from any other OECD country. (The OECD is just a for-example term: the world market now counts many non-OECD members.)

returns are related to risk in an efficient portfolio. This relationship is due to Sharpe (1964), Lintner (1965), and Mossin (1965).

19.2.1 How Asset Returns Determine the Portfolio Return

The model is typically derived in terms of returns rather than prices: academics use returns in empirical work, and practitioners want a formula for the expected return to be used for NPV applications. The key relation is that the realized return on the portfolio (subscript p) can always be written as (i) the risk-free return over that period, plus (ii), for all risky assets in the portfolio, the weighted average of the returns over and above the risk-free rate:

$$\tilde{r}_p - r = \sum_{j=1}^{N} x_j(\tilde{r}_j - r), \tag{19.7}$$

with a weight x_j defined as the initial amount invested in asset j, divided by total initial investment. A return over the risk-free rate is called an *excess return*, and its expected value is called the *risk premium.*

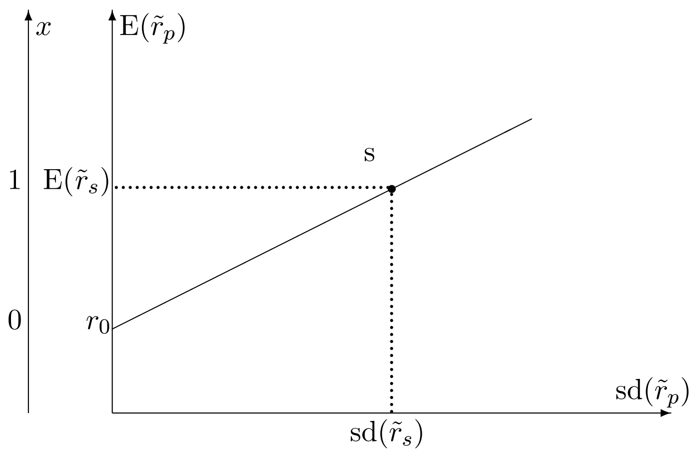
Example 19.2

You have 1000 to invest. Below, we show for three risky assets (denoted as 1, 2, 3) an initial price, the number of shares you buy, your total initial investment per asset, the asset weight, a possible time-1 price, the corresponding return, and the weighted return. The risky assets take up 900 of the money, so the balance, 100, is invested risk-free at, say, 5 percent. In the table we see the weights,[4] and how they sum to unity. We next compute the value of the portfolio at time 1, and see that it has gone up to 1105, implying a (net rate of) return of 0.105, *i.e.* 10.5 percent. The excess return is $10.5 - 5 = 5.5\%$, and this is exactly what you get by summing the value-weighted "excess" returns on the three risky assets.

		time-0 data and decisions				time-1 result		(excess) rates of return		
	j	$V_{j,0}$	n_j	$n_j V_{j,0}$	x_j	$V_{j,1}$	$n_j V_{j,1}$	r_j	$r_j - r$	$x_j(\tilde{r}_j - r)$
risky:	1	100	4	400	0.40	120	480	0.20	0.15	0.060
	2	50	4	200	0.20	70	280	0.40	0.35	0.070
	3	25	12	300	0.30	20	240	-0.20	-0.25	-0.075
subtotal				=900	=0.90					=0.055
risk-free	0			+100	+0.10		105			+0.05
total				=1000	=1.00		=1105			r_p=0.105

[4]Note that the weights we need in the formula are initial weights, determined by time-0 numbers, meaning that they are not stochastic.

Figure 19.1: **Combinations of risky stock portfolio s and asset 0**



DoItYourself problem 19.2

Rework the example by changing the initial investment in asset 1 from 400 to 800, maintaining the other risky positions and adjusting the risk-free one. Check that the weighted excess-return formula still gives the right answer.

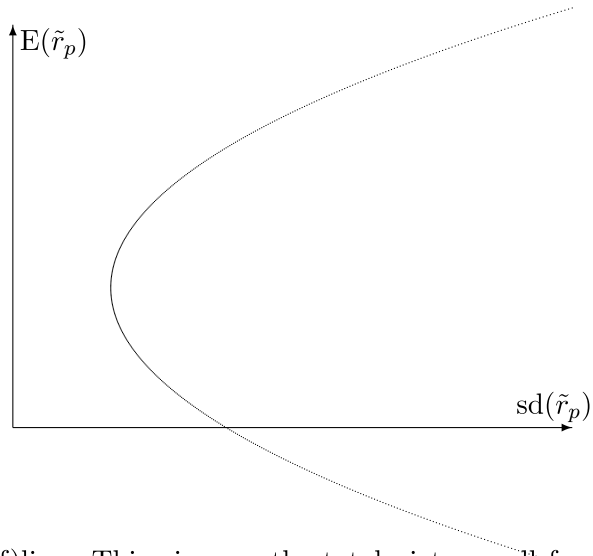
19.2.2 The Tangency Solution: Graphical Discussion

Consider the feasible combinations of expected return and standard deviation. The simplest case is one with a risk-free asset, subscripted "0", and a risky stock denoted as s. We invest a fraction x into the risky stock portfolio with return $\tilde{r}_s$ while $1-x$ is invested risk-free. The portfolio return is

$$\tilde{r}_p = x\tilde{r}_s + (1-x)r_0 = r_0 + x\,(\tilde{r}_s - r_0) \;\Rightarrow \begin{cases} \mathrm{E}(\tilde{r}_p) = r_0 + x\,\mathrm{E}(\tilde{r}_s - r_0), \\ \mathrm{sd}(\tilde{r}_p) = |x|\,\mathrm{sd}(\tilde{r}_s) \end{cases} \tag{19.8}$$

So for non-negative values of x, both expected return and standard deviation are linear functions of x. This will imply that all (E, sd) combinations achievable with the risk-free asset and the risky portfolio are found on a halfline. To show this we use a trick that is often applied in thermometers, where heat is measured on two scales, say Celsius and Fahrenheit, that are linearly related. Same here: x and $\mathrm{E}(\tilde{r}_p)$ are linearly related, so we can show them as two scales on one axis, as we do in Figure 19.1. To link the x and E_p scales we calibrate them using any two known corresponding points: $x = 1$ means $\mathrm{E}(\tilde{r}_p) = \mathrm{E}(\tilde{r}_s)$ while $x = 0$ means $\mathrm{E}(\tilde{r}_p) = r_0$. All this gives us the double-scaled axis shown in Figure 19.1. If $\mathrm{sd}(\tilde{r}_p)$ is linear in x_+, then looking at the other scale of the axis we must conclude it is linear in $\mathrm{E}(\tilde{r}_p)_{>r_0}$ too. The sd values for the calibration points are 0 and $\mathrm{sd}(\tilde{r}_s)$, respectively, and all risk-return combinations for intermediate or higher values of x or $\mathrm{E}(\tilde{r}_p)$ are on one

Figure 19.2: **The risk-return bound with just risky assets**



and the same (half)line. This gives us the total picture: all feasible combinations with $x \geq 0$ are on a halfline from $(\text{sd}(\tilde{r}_p), \text{E}(\tilde{r}_p)) = (0, r_0)$ through $(\text{sd}(\tilde{r}_s), \text{E}(\tilde{r}_s))$. The slope of that halfline is called the *Sharpe Ratio*:

$$\forall x \geq 0: \ \frac{\text{E}(\tilde{r}_p - r_0)}{\text{sd}(\tilde{r}_p)} = \frac{\text{E}(\tilde{r}_s - r_0)}{\text{sd}(\tilde{r}_s)} = s\text{'s Sharpe Ratio.} \tag{19.9}$$

Now look at a second simple case, where the portfolio consists of two imperfectly correlated risky assets, subscripted 1 and 2. Now we have

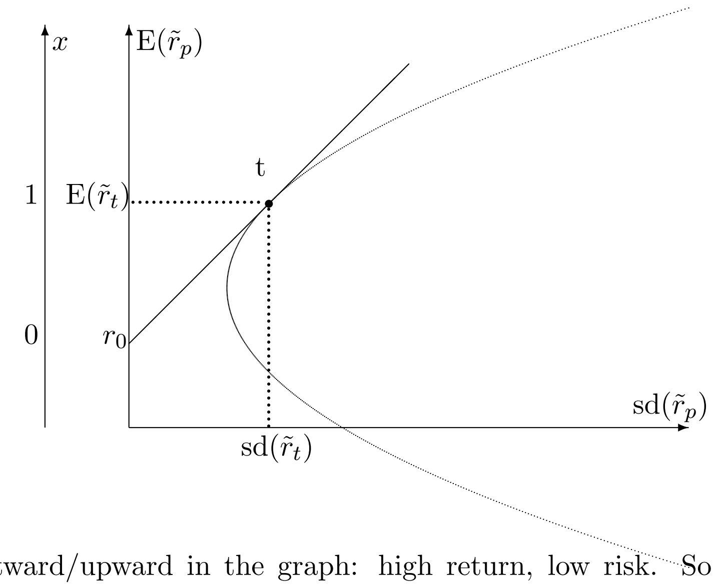
$$\tilde{r}_p = x_1\tilde{r}_1 + (1 - x_1)\tilde{r}_2; \tag{19.10}$$

$$\Rightarrow \quad \begin{cases} \text{E}(\tilde{r}_p) = \text{E}(\tilde{r}_1) + x_1\,[\text{E}(\tilde{r}_1) - \text{E}(\tilde{r}_2)], \\ \\ \text{sd}(\tilde{r}_p) = \sqrt{x_1^2\text{var}(\tilde{r}_1) + 2x_1(1-x_1)\text{cov}(\tilde{r}_1, \tilde{r}_2) + (1-x_1)^2\text{var}(\tilde{r}_2)} \end{cases} \tag{19.11}$$

From the first implication we conclude that x_1 and expected return are still two sides of the same thermometer. The sd function looks messier. But we immediately see that variance is quadratic in x_1 and, therefore, in $\text{E}(\tilde{r}_p)$ too. This means a rotated "U"-shape-like graph (or a rounded V, if you want) opening towards the right. Warping the risk axis by taking squareroots does not fundamentally change the shape of the relation, as you can check using a spreadsheet. We end up with a feasible set like in Figure 19.2. Basic textbooks will tell you that, if there are more than two risky assets, the feasible combinations in a (std, E) space graph is still similar.

The last step is to look at N risky assets and a risk-free one. We return to Figure 19.1 except that the risky part of the portfolio, s, must be chosen from a feasible set shaped like in Figure 19.2. A risk-averse mean-variance investor wants

Figure 19.3: **Efficient Portfolios & the Tangency Portfolio**



to be leftward/upward in the graph: high return, low risk. So s will be chosen from the left-upper risky-asset bound. Among all such portfolios, the best one is the portfolio that rotates the halfline from $(\mathrm{sd} = 0, \mathrm{E} = r_0)$ as far upward/leftward as is feasible—the one with the highest Sharpe Ratio. It follows that the optimal choice is the *tangency portfolio*, the one where the halfline from $(\mathrm{sd} = 0, \mathrm{E} = r_0)$ just touches the V-curve that bounds the risky-assets risk-return set. All portfolios on this halfline are efficient. They all are combinations of the risk-free asset and the *tangency portfolio*, subscripted t.

We now want to take a peek at the analytical solution and its implication. To understand how the tangency portfolio can be found we need to understand first how a small change in one of the portfolio weights affects the expected return and the variance of the portfolio return.

19.2.3 How Portfolio Choice Affects Mean and Variance of the Portfolio Return

We want to understand what happens if investors choose portfolios on the basis of the mean and variance of the portfolio return. To figure out how these people think, we need to understand how portfolio choice affects the mean and variance of the total return. The link is, of course, Equation [19.7]: $\tilde{r}_p = r + \sum_{j=1}^{N} x_j\,(\tilde{r}_j - r)$. From this it follows that

$$\mathrm{E}(\tilde{r}_p) = r + \sum_{j=1}^{N} x_j\,\mathrm{E}(\tilde{r}_j - r), \qquad (19.12)$$

$$\mathrm{var}(\tilde{r}_p) = \sum_{j=1}^{N} x_j \sum_{k=1}^{N} x_k\,\mathrm{cov}(\tilde{r}_j,\,\tilde{r}_k). \qquad (19.13)$$

The first formula is pretty obvious. To interpret the second one, it helps to derive it in two steps, as follows:[5]

$$\mathrm{var}(\tilde{r}_p) = \mathrm{cov}(\tilde{r}_p,\, \tilde{r}_p) = \mathrm{cov}(\sum_{j=1}^{N} x_j\tilde{r}_j,\, \tilde{r}_p) = \sum_{j=1}^{N} x_j\,\mathrm{cov}(\tilde{r}_j,\, \tilde{r}_p), \quad (19.14)$$

where

$$\mathrm{cov}(\tilde{r}_j,\, \tilde{r}_p) = \mathrm{cov}(\tilde{r}_j,\, \sum_{k=1}^{N} x_k\tilde{r}_k) = \sum_{k=1}^{N} x_k\,\mathrm{cov}(\tilde{r}_j,\, \tilde{r}_k). \quad (19.15)$$

This tells you that the portfolio variance is a weighted average of each asset's covariance with the entire portfolio; and each of these assets' portfolio covariances is, in turn, a weighted average of the asset's covariance with all components of the portfolio.

Example 19.3

We compute the portfolio expected excess return, the assets' covariances with the portfolio return, and the portfolio variance when the risky assets' weights are 0.50 and 0.40 (implying $x_0 = 0.10$):

	$\mathrm{E}(\tilde{r}_j - r)$	$\mathrm{cov}(\tilde{r}_j, \tilde{r}_1)$	$\mathrm{cov}(\tilde{r}_j, \tilde{r}_2)$
1	0.200	0.16	0.05
2	0.122	0.05	0.09

$$\begin{aligned}
\mathrm{E}(\tilde{r}_p - r) &= 0.50 \times 0.200 + 0.40 \times 0.122 = 0.1488 \\
\mathrm{cov}(\tilde{r}_1, \tilde{r}_p) &= 0.50 \times 0.160 + 0.40 \times 0.050 = 0.1000 \\
\mathrm{cov}(\tilde{r}_2, \tilde{r}_p) &= 0.50 \times 0.050 + 0.40 \times 0.090 = 0.0610 \\
\Rightarrow \mathrm{cov}(\tilde{r}_p, \tilde{r}_p) &= 0.50 \times 0.100 + 0.40 \times 0.061 = 0.0744
\end{aligned}$$

How do these numbers change when the portfolio weights are being tweaked? First look at a two-risky-assets example and see how mean and variance are affected by a small change in the weight of asset 1 (implicitly matched by a small offsetting change in the weight for the risk-free bond, asset zero):

$$\begin{aligned}
\mathrm{E}(\tilde{r}_p - r) &= x_1\,\mathrm{E}(\tilde{r}_1 - r) + x_2\,\mathrm{E}(\tilde{r}_2 - r); \\
\Rightarrow \frac{\partial \mathrm{E}(\tilde{r}_p - r)}{\partial x_1} &= \mathrm{E}(\tilde{r}_1 - r), \quad (19.16)
\end{aligned}$$

[5] We use the fact that, inside a variance, risk-free returns added or subtracted play no role: $\mathrm{var}(r + \sum x_j(\tilde{r}_j - r)) = \mathrm{var}(\sum x_j\tilde{r}_j)$.

and

$$\begin{aligned} \mathrm{var}(\tilde{r}_p) &= x_1^2\mathrm{var}(\tilde{r}_1) + 2x_1x_2\mathrm{cov}(\tilde{r}_1,\tilde{r}_2) + x_2^2\mathrm{var}(\tilde{r}_2); \\ \Rightarrow \frac{\partial \mathrm{var}(\tilde{r}_p)}{\partial x_1} &= 2x_1\mathrm{var}(\tilde{r}_1) + 2x_2\mathrm{cov}(\tilde{r}_1,\tilde{r}_2), \\ &= 2[x_1\mathrm{cov}(\tilde{r}_1,\tilde{r}_1) + x_2\mathrm{cov}(\tilde{r}_1,\tilde{r}_2)], \\ &= 2[\mathrm{cov}(\tilde{r}_1,x_1\tilde{r}_1) + \mathrm{cov}(\tilde{r}_1,x_2\tilde{r}_2)], \\ &= 2\mathrm{cov}(\tilde{r}_1,x_1\tilde{r}_1 + x_2\tilde{r}_2), \\ &= 2\mathrm{cov}(\tilde{r}_1,\tilde{r}_p). \end{aligned} \tag{19.17}$$

Similarly, $\frac{\partial \mathrm{E}(\tilde{r}_p - r_0)}{\partial x_2} = \mathrm{E}(\tilde{r}_2 - r)$ and $\frac{\partial \mathrm{var}(\tilde{r}_p)}{\partial x_2} = 2\mathrm{cov}(\tilde{r}_2,\tilde{r}_p)$.

DoItYourself problem 19.3

Recompute the expected excess return and variance when, in the previous example, x_1 is increased from 0.50 to 0.51. Check how the scaled change in the mean, $\Delta\mathrm{E}/\Delta x_1$, is exactly the first asset's own expected excess return. Likewise, check how the scaled change in the variance, $\Delta\mathrm{var}/\Delta x_1$, is about twice the first asset's own covariance with the portfolio return.[6]

DoItYourself problem 19.4

Consider a portfolio with, initially, $x_1 = 0.5$ and $x_2 = 0$ so that $\mathrm{var}(\tilde{r}_p) = \mathrm{var}(0.5\,\tilde{r}_1) = 0.5^2\,\mathrm{var}(\tilde{r}_1)$. Then increase the second weight to 0.001. Write out the change in the variance, and check whether it is far from $2\mathrm{cov}(\tilde{r}_2,\tilde{r}_p) \times 0.001$.

19.2.4 Efficient Portfolios: A Review

Recall that a portfolio is efficient if it has the highest expected return among all conceivable portfolios with the same variance of return. We just reviewed the probably familiar result that any efficient portfolio is a combination of two building blocks: the risk-free asset, and the tangency portfolio of risky assets (Figure 19.3). But what is perhaps less obvious is how the tangency portfolio must be constructed and what this implies for the risk-return relation. Let us consider this.

It is easily shown that, if a portfolio is to be efficient, then for each and every asset the marginal risk-return ratio—the ratio of any asset's marginal "good" (its contribution to the portfolio's expected excess return) to the asset's marginal "bad"

[6] For the variance, the scaled difference is not perfectly the same as the partial derivative because the function is quadratic in the weights, not linear. (For non-linear functions, obviously, $\Delta y/\Delta x \neq \mathrm{d}x/\mathrm{d}y$.) But note how the scaled change in fact equals the average of the original and the revised covariances (0.1000 when $x_1 = 0.50$, and 0.1016 when $x_1 = 0.51$). In the limit, the two covariances are so close that they become indistinguishible from their average.

(its contribution to the portfolio's risk)—must be the same, see TekNote 19.1. We just identified the asset's contribution to the portfolio's expected excess return as the asset's own expected excess return, while the asset's contribution to the portfolio variance is twice the covariance between the asset's return and the portfolio return. Thus, the general efficiency condition can be written as follows:

$$\frac{E(\tilde{r}_j - r)}{cov(\tilde{r}_j, \tilde{r}_p)} = \lambda, \text{ for all risky assets } j\text{=1, ... N}, \tag{19.18}$$

where r is the risk-free rate of return, and $\tilde{r}_j$ the uncertain return on asset j. The common return/risk ratio, λ, depends on the investor's attitude toward risk, and is called the investor's *relative risk aversion.*

Example 19.4

Let there be two risky assets (j = 1, 2), with the following expected excess returns and covariances of return:

	$E(\tilde{r}_j - r)$	(co)variances	
Asset 1	0.092	$\text{cov}(\tilde{r}_1, \tilde{r}_1) = 0.04$	$\text{cov}(\tilde{r}_1, \tilde{r}_2) = 0.05$
Asset 2	0.148	$\text{cov}(\tilde{r}_2, \tilde{r}_1) = 0.05$	$\text{cov}(\tilde{r}_2, \tilde{r}_2) = 0.09$

Given these data, a portfolio p with weights ($x_1 = 0.4$, $x_2 = 0.6$) is efficient. We can verify the efficiency of this portfolio in two steps:

- First we compute the contribution of each asset to the total risk of portfolio p (covariance), as follows:[7]

  $$\begin{array}{ll} \text{Asset 1:} & \text{cov}(\tilde{r}_1, x_1\tilde{r}_1 + x_2\tilde{r}_2) = 0.4 \times 0.04 + 0.6 \times 0.05 = 0.046, \\ \text{Asset 2:} & \text{cov}(\tilde{r}_2, x_1\tilde{r}_1 + x_2\tilde{r}_2) = 0.4 \times 0.05 + 0.6 \times 0.09 = 0.074. \end{array}$$

- Next we compute, for each asset, the excess return/risk ratio and note that both ratios equal 2:

  $$\frac{0.092}{0.046} = 2 = \frac{0.148}{0.074}, \tag{19.19}$$

  which implies that the portfolio is efficient.

Moreover, this is not just any efficient portfolio: it actually is the tangency portfolio of risky assets. This is because (1) any efficient portfolio is a combination of the risk-free asset and the tangency portfolio of risky assets, and (2) this particular efficient portfolio contains no risk-free assets.

[7]We use the fact that the return on the risk-free asset does not co-vary with any risky asset's return.

The portfolio in the example will be selected by an investor with relative risk aversion equal to $\lambda = 2$. One way to detect differences in risk aversion among mean-variance investors is to watch the proportions they invest in the risk-free asset. An investor with a higher relative risk aversion simply allocates more of his or her wealth to the risk-free asset, and less to the tangency portfolio of risky assets.

Example 19.5

Suppose that an investor invests half of his or her wealth in the tangency portfolio identified in the previous example, and the remainder in the risk-free asset. That is, the weights in portfolio p' are $x_0 = 0.5$ for the risk-free asset, and ($x_1 = 0.2$, $x_2 = 0.3$) for the risky assets. We can easily verify that p' is still an efficient portfolio and that this investor has a relative risk aversion equal to 4:

- The risks of the assets in portfolio p' are computed as follows:

$$\begin{aligned} &\text{Asset 1:} \quad \text{cov}(\tilde{r}_1,\, x_1\tilde{r}_1 + x_2\tilde{r}_2) = 0.2 \times 0.04 + 0.3 \times 0.05 = 0.023, \\ &\text{Asset 2:} \quad \text{cov}(\tilde{r}_2,\, x_1\tilde{r}_1 + x_2\tilde{r}_2) = 0.2 \times 0.05 + 0.3 \times 0.09 = 0.037. \end{aligned}$$

- The excess return/risk ratios now both equal 4:

$$\frac{0.092}{0.023} = \frac{0.148}{0.037} = 4. \tag{19.20}$$

  which implies that the portfolio is also efficient.

Thus, the investor's relative risk aversion can be inferred from his or her portfolio choice. Relative to the tangency portfolio chosen by an investor with $\lambda = 2$, the more risk-averse investor with $\lambda = 4$ simply reduces the proportion invested in the risky assets by half. This, as we notice, also halves the (covariance) risks of each risky asset in the total portfolio. This stands to reason: if the total portfolio risk falls, assets' contributions to that total risk must fall too.

There is another, related, way to measure risk aversion: compute the excess-return-to-variance ratio for the entire portfolio. This ratio produces the same number as the previous ones since it takes the same linear combination of both numerators and denominators:[8]

$$\text{if } \frac{0.092}{0.023} = \frac{0.148}{0.037} = 4 \text{ then } \frac{0.2 \times 0.092 + 0.3 \times 0.148}{0.2 \times 0.023 + 0.3 \times 0.037} = 4. \tag{19.21}$$

We conclude that, for efficient portfolios, the holder's relative risk aversion can be measured by the overall excess-return/risk ratio:

$$\text{Relative risk aversion } = \lambda = \frac{\text{E}(\tilde{r}_p - r)}{\text{var}(\tilde{r}_p),}, \tag{19.22}$$

[8] The general way to establish this is to write the efficiency condition as $\text{E}(\tilde{r}_j - r) = \lambda\, cov(\tilde{r}_j,\, \tilde{r}_p)$. This implies $x_j\text{E}(\tilde{r}_j - r) = \lambda\, x_j\text{cov}(\tilde{r}_j,\, \tilde{r}_p)$ and therefore $\sum_j x_j\text{E}(\tilde{r}_j - r) = \lambda \sum_j x_j\text{cov}(\tilde{r}_j,\, \tilde{r}_p) = \lambda\, \text{cov}(\sum_j x_j\tilde{r}_j,\, \tilde{r}_p)$. Thus, $\text{E}(\tilde{r}_p - r) = \lambda\, cov(\tilde{r}_p,\, \tilde{r}_p) = \lambda\, \text{var}(\tilde{r}_p)$.

a relation that comes in good stead to derive the CAPM in the next subsection.

Using a variety of proxies for the market portfolio and a variety of methodologies, [19.22] has been used to estimate the US average risk aversion. The estimates vary, but the consensus in long-term tests is that λ exceeds unity. Also this result will come in handy later.

19.2.5 The Market Portfolio as the Benchmark

Let us now go from an individual investor's portfolio to the market portfolio—defined as the aggregate asset holdings of all investors in a particular group. The group typically considered in the standard CAPM is composed of all investors in the economy. What exactly "the" economy corresponds to in practice—a country? a region?—is left vague, but, crucially, this set of investors is assumed to have *homogeneous opportunities,* that is, equal access to the same list of assets, and *homogeneous expectations*, that is, equal perceptions about the return characteristics of the assets.

The effect of these homogeneity assumptions is that all of the investors agree about the composition of the tangency portfolio. If each investor holds the risk-free asset plus the same tangency portfolio, then also the aggregate portfolio must be a combination of the risk-free asset plus that very same tangency portfolio. But any such combination is efficient. Therefore, for the market portfolio (denoted by subscript m), the efficiency condition Equation [19.1] must hold, with λ_m now defined as the market's risk aversion (which can be shown to be a kind of weighted average of the individuals' risk aversions):

$$\frac{E(\tilde{r}_j - r)}{\mathrm{cov}(\tilde{r}_j, \tilde{r}_m)} = \lambda_m, \text{ for all risky assets } j\text{=1, ... N.} \tag{19.23}$$

Although Equation [19.23] is not yet written in the standard CAPM form, this equation already is an embryonic capital asset pricing model because it tells us what the expected excess return should be as a function of the asset's covariance risk in the market portfolio. To implement the model, we need to know the relative risk aversion for the average investor. But we just found a way to infer this: just use [19.22] to identify the market's relative risk aversion. This leads us straight to the CAPM:

$$\begin{aligned} \mathrm{E}(\tilde{r}_j - r) = \lambda_m \,\mathrm{cov}(\tilde{r}_j, \tilde{r}_m) &= \frac{\mathrm{E}(\tilde{r}_m - r)}{\mathrm{var}(\tilde{r}_m)} \mathrm{cov}(\tilde{r}_j, \tilde{r}_m) \\ &= \beta_{j,m}\, \mathrm{E}(\tilde{r}_m - r), \end{aligned} \tag{19.24}$$

In Equation [19.24], $\beta_{j,m} = \mathrm{cov}(\tilde{r}_j,\, \tilde{r}_m)/\mathrm{var}(\tilde{r}_m)$ is the asset's rescaled covariance risk, or the asset's beta. The advantage of rescaling the covariance risk is that $\beta_{j,m}$ is also the slope coefficient from the so-called *market model*, the regression of the return from asset j, on the return from the market portfolio, $\tilde{r}_j = \alpha_{j,m} + \beta_{j,m}\tilde{r}_m + \epsilon_{j,m}$.

Thus, the rescaled risk (the asset's relative risk, or market sensitivity) in Equation [19.24] can be estimated using time-series data of past stock returns and market returns, assuming, at least, that beta risks and expected returns are constant. We can summarize this model as follows:

- The beta is a measure of the asset's relative risk—that is, the asset's market covariance risk $\text{cov}(\tilde{r}_j, \tilde{r}_m)$, rescaled by the portfolio's total risk, $\text{var}(\tilde{r}_m)$. Beta can be estimated from the market-model regression.

- A risky asset with beta equal to zero should have an expected return that is equal to the risk-free rate, even if the asset's return is uncertain. The reason is that the marginal contribution to the total market risk is zero.

- If an asset's beta or relative risk is non-zero, the asset's expected return should contain a risk premium. The additional return that can be expected per unit of beta is the market's expected excess return above the risk-free rate.

19.2.6 A Replication Interpretation of the CAPM

An enlightening joint interpretation of the market model regression and the CAPM is as follows. A regression $\tilde{y} = a + b\tilde{x} + \tilde{e}$ has the property that it offers the best possible fit between $\tilde{y}$ and $a + b\tilde{x}$, in the sense that no other numbers a and b produce a lower residual variance, $\text{var}(\tilde{e})$. Now suppose that you were asked to find a combination of investments in the risk-free asset and a market-index fund that best resembles a particular asset, say Apple Computer common stock. This best-replication portfolio can be identified by regressing Apple's return onto the market return:

Example 19.6

Suppose that $\beta_{Apple} = 0.75$. If we invest 75 percent in the market and 25 percent in the risk-free asset, we hold a portfolio that offers the best possible replication of Apple Computer's return, among all portfolios that consist only of the market portfolio and the risk-free asset.

As, in the best replication, a fraction β is invested in the market and $(1 - \beta)$ in the risk-free asset, the expected return on such a best-replication portfolio would be

$$\begin{aligned} \text{E}(\tilde{r}_{Apple's\,replication}) &= \beta_{Apple}\,\text{E}(\tilde{r}_m) + (1 - \beta_{Apple})r \\ &= r + \beta_{Apple}\text{E}(\tilde{r}_m - r). \end{aligned} \tag{19.25}$$

But this is exactly the CAPM's prediction of the return on Apple itself. So the CAPM tells us that the expected return on stock j is equal to the expected return on its best replicating portfolio.

In that sense the logic of the CAPM is to some extent similar to the logic of asset-pricing-by-replication, as used in Part II of this book, except that we now use the best possible replication rather than exact replication. Because the replication is not exact, we need the CAPM assumptions to justify why the expected return on an asset should still be the same as the expected return on its best replicating portfolio, and why the market portfolio is the only replication instrument that is to be considered. In the CAPM logic, investors do not care about the imperfections in the replication (that is, the part of Apple's return not "explained" by the market) because they all hold the market portfolio anyway; the part of Apple's return not correlated with $\tilde{r}_m$ is simply diversified away.

19.2.7 When to Use the Single-Country CAPM

The CAPM as derived in Section 19.2 is routinely used in capital budgeting to determine the return that shareholders expect on investments with a given level of beta risk. For many countries, financial institutions provide estimates of the betas for various industries. Yet, one ought to interpret these figures with some caution. The assumption that underlies many of these estimates is that the CAPM holds country-by-country, in the sense that the market portfolio is equated with the portfolio of all assets issued by firms from that country alone. For example, beta service companies in the US tend to compute the beta of, say, the US computer industry by regressing the returns from a portfolio of US computer firms on the Vanguard index, which is an index of thousands of US stocks traded on the New York Stock Exchange, Amex, and NASDAQ. Likewise, in France, one would often estimate the risk of, say, the French steel industry by regressing the returns from a portfolio of steel companies on the index of French stocks. In the same vein, the expected excess return on the market would be estimated from past returns on the Vanguard index or on the index of French stocks traded at the Paris section of Euronext, respectively.

Is the market portfolio of assets held by a country's investors the same as the portfolio of assets issued by the country's corporations? This is only true if investors have access to local shares only *and* all local shares are held by residents of the country. That is, if one equates the market portfolio with the portfolio of locally issued shares, capital markets are assumed to be fully segmented. However, in most countries there are no rules against international share ownership; investors can easily diversify into foreign assets, and foreigners are allowed to buy domestic shares. Thus, the traditional interpretation that the market portfolio consists of the index of stocks issued by local companies is valid only in segmented markets.

Example 19.7

Until the later 1990s, the stock markets of India, South Korea, and Taiwan were almost perfectly segmented from the rest of the world in the sense that foreigners could buy only a small fraction of the local stocks, and local investors could not easily buy foreign assets. Thus, the Indian market portfolio was essentially the same as the portfolio of stocks issued by Indian firms, and similar for Korea and Taiwan.

In the presence of market segmentation, the cost of capital to be used by, say, a

North-American or European firm is likely to be different from the cost of capital to be used by an Indian firm, even when these companies are evaluating similar investments. For the Indian case, we would have used a one-country CAPM. The question addressed in the next section is how, say, a Canadian firm should determine its cost of capital, knowing that its investors are part of a market that is much wider than just Canada. There are no rules that prevent Canadian investors from buying US or European assets, nor are nonresidents barred from buying Canadian stocks. Under these circumstances the index of stocks issued by Canadian firms is likely to be a poor proxy for the portfolio held by the average Canadian investor. It follows that a Canadian firm cannot use the single-country CAPM to set the cost of capital for an investment project. Not only does the Canadian-stock index miss foreign stocks held by residents, but it also ignores the fact that many Canadian stocks are held by foreigners. Note also that this problem arises whether the project is domestic or foreign: it's not as if Canadians can still use a one-country CAPM for home investments, and only have a problem if the project is foreign.

19.3 The International CAPM

As we just stated, there are no rules preventing Canadian investors from buying US or European assets; nor are there any regulations barring nonresidents from buying Canadian stocks. Still, this mere fact is not sufficient to lead to international diversification by investors. We have already argued, in Chapter 18, that there are strong incentives for investors to diversify internationally. We just pointed out why this causes a problem with the standard CAPM, at least in the version that uses the locally-issued stock index rather than the locally-held stock index. From this starting point we add four items: we explain the role of exchange risk for asset pricing in an internationally integrated capital market; we derive a two-country version of the International CAPM of Solnik (1973) and Sercu (1980, 1981); we generalize to the case with many countries and stochastic inflation; and we conclude with a review of empirical tests of the International CAPM.

19.3.1 International diversification and the traditional CAPM

International diversification is beneficial for the investor, and investors do use this added opportunity to reduce risks. Clearly, it is then no longer acceptable to use a CAPM equation with, as its benchmark portfolio, the local stock index (defined as the index of all securities issued by firms incorporated in the country). First, this benchmark omits foreign assets, which represent an important component of the local investor's asset holdings. Second, this benchmark ignores the fact that a substantial part of the stocks issued by local corporations are, in fact, held by nonresidents.[9] All of this means that, in internationally integrated markets, the true stock market portfolio for any country is unobservable—and, with an unobservable national stock market portfolio, the standard CAPM is of no practical use to managers who, for instance, want to assess the cost of capital or evaluate the performance of their investment advisers.

19.3.2 Why Exchange Risk Pops up in the International Asset Pricing Model

How can we get around this problem of an unobservable market portfolio? One could argue that, even if we do not know what shares are held by whom, we can still observe the *world* market portfolio. (For conciseness, we will refer to the the countries that allow free capital movements as "the world", with an apology to residents from China and other unworldly countries.) Even if we do not know what stock is held by whom and where, we do know what stocks are listed somewhere in the world and how many shares are outstanding at what price. Thus, the world market portfolio contains all securities issued by all firms in the world, and it can be obtained by constructing a value-weighted sum of all member countries' local stock indices.[10] As investors do hold assets from all over the world, and as the world market portfolio is observable, a very simple approach to international asset pricing would be to interpret the world as one huge country, and use the world market portfolio as the benchmark in a unified-world CAPM.

There is, however, one important reason why international asset pricing in integrated capital markets cannot simply be reduced to an as-if-one-country CAPM. Even if international capital transactions are unrestricted and have low costs, transactions in the commodity markets are still difficult and costly. These imperfections in the goods market, as we saw in Chapter 3, lead to substantial deviations from relative purchasing power parity and to real exchange risk. The (real) return on, say, IBM common stock as realized by a German investor differs from the (real) return realized by a Japanese investor on the same asset. As a result, the distributions of the real return from a given asset depend on the nationality of the investor. This then violates the homogeneous expectations assumption of the CAPM, which states that all investors agree on the probability distribution of the (real) asset returns. In a sense, the investors' perceptions about real return distributions are segmented along country lines because goods prices differ across countries, implying that investors

[9]The same problem arises when one includes into the market portfolio all stocks—domestic or foreign—that are listed on the national stock exchange(s). Investors can (and do) buy foreign assets in foreign stock exchanges, or can (and do) buy foreign assets through mutual funds that are traded over-the-counter; and all of these investments are missing from the menu of locally listed stocks.

[10]A well-known proxy for such an international stock market index is the Morgan Stanley Capital International (MSCI) index, or Datastream's World Market Index. Both are biased towards large firms; but small firms are held locally, mostly, so that's not a huge problem.

from various countries have different views on the distributions of real returns on any given asset or portfolio.

Example 19.8

A clear example is the return on the two countries' T-bills. Suppose that there is no inflation. While to a US investor, the CAD T-bill is one of the available risky assets, it is risk-free to a Canadian investor. On the other hand, the USD T-bill is a risky asset to a Canadian investor but risk-free to a US investor. Thus, the perceived distribution of (real) returns depends on the nationality of the investor.

Thus, we need to derive a CAPM that takes into account the heterogeneous viewpoints of investors from various countries. In keeping with our discussion of the standard CAPM, we initially ignore inflation. To simplify the analysis, we shall initially assume there are just two countries, the US and Canada. Once you understand the two-country model, you can easily generalize to the case of many countries.

The problem is that the Canadian investor's portfolio choice is based on how each asset contributes to the variance and expected excess return on the portfolio measured in CAD, while the US investor's portfolio choice is based on the assets' contributions to a portfolio whose risk and return are expressed in USD. Let, as usual, the asterisk refers to the foreign country (say, the US); p^* refers to the portfolio held by the US investor; $\tilde{r}_j^*$ refers to a return in FC on stock j (whose nationality, if any, we do not really need to know); r^*, unsubscripted, as usual refers to the USD risk-free rate; and $\tilde{r}_{p^*}^*$ denotes the return, in USD, on the US market portfolio p^*. Then what we know about portfolio choice can be summarized as follows:[11]

$$\text{Canadians choose } p \text{ such that } \mathrm{E}(\tilde{r}_j - r) = \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_p), \tag{19.26}$$

$$\text{Americans choose } p^* \text{ such that } \mathrm{E}(\tilde{r}_j^* - r^*) = \lambda \operatorname{cov}(\tilde{r}_j^*, \tilde{r}_{p^*}^*). \tag{19.27}$$

What, then, is the relation between expected excess returns and the world market portfolio, which is the sum of p and p^*? To identify that link, we have to translate [19.27] into the same currency as [19.26], the CAD. Using a trick called Ito's Lemma (see Technical Note 19.2), [19.27] can be translated into CAD as follows:

$$\text{Americans choose } p^* \text{ such that } \mathrm{E}(\tilde{r}_j - r) = \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1-\lambda) \operatorname{cov}(\tilde{r}_j, \tilde{s}), \tag{19.28}$$

where $\tilde{s}$ is the percentage change in the exchange rate (CAD per USD). What is going on here is that US investors really care about their wealth expressed in USD, W_{us}^*, because the consumption prices relevant to them are (almost) constant in USD and far less so in CAD. We can always re-express W_{us}^* as CAD-measured wealth divided by the CAD/USD exchange rate, $W_{us}^* = W_{us}/S$. So people who care about W^* will act as if they care about wealth in CAD sure enough—because, everything else

[11] It would not have been very painful to allow for different risk aversions across countries too, but little additional insight would have been gained, so we set $\lambda^* = \lambda$.

Table 19.2: **Exchange Rate Exposure: Good or Bad?**

	Example 1: covariance > 0		Example 2: covariance < 0		Comment
W_{us} (in CAD)	12,000	16,000	12,000	16,000	same distribution for W_{us} ...
S (CAD/USD)	1.00	1.50	1.50	1.00	and same distribution for S;
W_{us}^* (in USD)	12,000	10,667	8,000	16,000	but the positive-cov case has ...
$\mathrm{E}(W_{us}^*)$	11,333		12,000		– a lower mean W_{us}^* ...
stdev(W_{us}^*)	667		4,000		– and a lower stdev W_{us}^*

being the same, the higher their CAD wealth, the higher also their wealth in USD. The fact that, holding constant the exchange rate, they care about CAD-expressed wealth then explains why the first half of the efficiency condition looks like the Canadian investor's condition. But US investors will all the time also think of the exchange rate, because deep down they care about USD-measured wealth only. It is this concern about the exchange rate that induces a second item. But, as we shall see, it is less obvious whether the US investor, thinking in CAD terms but caring about USD numbers, likes exchange-rate exposure or not.

Example 19.9

In Table 19.2 we have picked two examples where, in each example, there are two equally probable scenarios for CAD wealth and the exchange rate. The means and variances are the same across the two examples, but the first one has a positive association between CAD wealth and the exchange rate while in the second example the correlation is negative. We see that a larger positive covariance is a mixed blessing: it lowers both the mean (bad!) and the variance (good!). So whether on balance the effect is preferred depends on your degree of risk aversion, notably whether you attach more weight to the rise in return than to the rise of risk.

It can, in fact, be shown that investors with risk aversion equal to 1 ignore covariance with S. More risk-averse investors ($\lambda > 1$) like it because they like the variance-reduction effect, while less risk-averse people dislike it: the drop in the mean is viewed as too high a price for the lower risk. But note that, among financial economists, the consensus probably is that lambda exceeds unity. (Macro-economists are not so sure.) Thus, the modal investor probably prefers the hedging effect and is willing to accept a lower mean return on asset j if it does help as a hedge.

What assets would be especially attractive to US investors from that perspective? One might guess that US stocks may be more appealing than Canadian stocks. But such a view may be simplistic, as the next subsection argues.

19.3.3 Do Assets have a Clear Nationality?

For a better understanding of the exchange rate covariance risk of individual assets, it is convenient to scale the covariance risk by the exchange rate variance. Consider the following regression equation:

$$\tilde{r}_j = \alpha_{j,s} + \gamma_j \, \tilde{s}_{\text{CAD/USD}} + \epsilon_{j,s}. \tag{19.29}$$

The regression coefficient γ_j equals $\text{cov}(\tilde{r}_j, \tilde{s})/\text{var}(\tilde{s})$—the asset's exchange rate covariance risk, scaled by the variance of the exchange rate change. In this sense, γ_j measures the relative exchange risk of asset j, or the relative exposure of asset j to the exchange rate, in the same way beta measures the relative exposure of a stock to market movements. We now consider the exchange rate exposure of six types of assets: a domestic and foreign risk-free asset, a foreign exporter and importer, and a domestic exporter and importer.

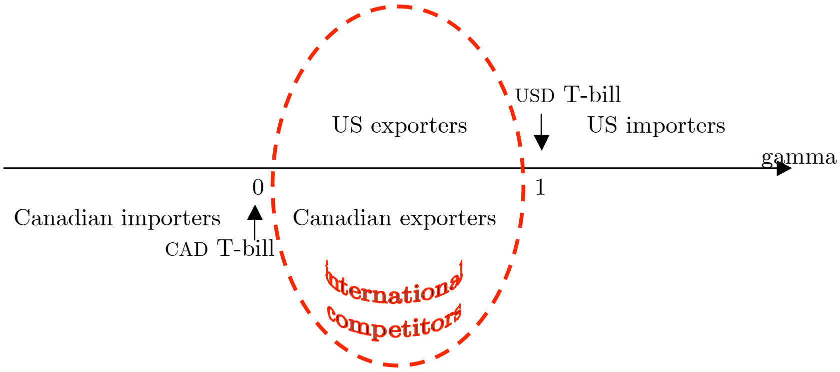
- Let us consider the domestic T-bill, asset 0. Since this return is not stochastic, it has zero exposure to the exchange rate.
- The next asset we consider is the USD T-bill, asset 1. The return, measured in CAD, on the USD T-bill increases by one percent if the CAD/USD spot rate increases by one percent. This follows from

  $$\tilde{r}_{\text{USD}\,Tbill} \approx r^* + \tilde{s}_{\text{CAD/USD}}. \tag{19.30}$$

  Clearly, if $\tilde{r}_j = r^* + \tilde{s}_{\text{CAD/USD}}$, then, in the relative exposure regression Equation [19.29], we must have $\gamma_{\text{USD}\,Tbill} = 1$ (and $\alpha_{\text{USD}\,Tbill} = r^*$). In this sense, the exposure regression (Equation [19.29]) for the foreign T-bill will reveal a very clear nationality for that asset. In CAD terms, the USD T-bill is exposed one-to-one to its "own" exchange rate, CAD/USD.

  Thus far, things are clear: the home T-bill has zero exposure and the foreign one has a unit exposure. For stocks, however, nationality is much more blurred:
- Let asset 2 be a Canadian importer. Typically an appreciation of the USD relative to the CAD is bad news for such Canadian firm, because its costs have gone up. Thus, for a Canadian importer, the relative exposure (γ_j) is negative.
- Let us now consider as asset 3 a Canadian producer competing against US producers in the US and/or Canadian market. Typically an appreciation of the USD relative to the CAD is good news for such a Canadian firm, because its competitive position has improved. Thus, for a Canadian exporter or import-substituter, the relative exposure (γ_j) is positive.
- The next case we look at is a US corporation that competes against Canadian firms in the US and/or Canadian market. Holding constant the USD price of the stock, a one percent appreciation of the USD adds one percent to the return on that stock in CAD. However, an appreciation of the USD simultaneously is

Figure 19.4: **Relative exposures (γ) of various assets**



bad news for this company, because its competitive position has deteriorated. Thus, the price of the stock measured in USD typically drops when the USD appreciates. This drop in the USD value of the stock weakens the effect of the exchange rate itself, and will lead to a relative exposure that is below unity.

Example 19.10

Suppose that, empirically, the stock price in USD of a US firm goes down by, on average, 0.25 percent for a 1 percent increase in the CAD/USD rate. This then implies that the return, in CAD, on the stock will go up by about 0.75 percent for a one percent rise in the CAD/USD rate. That is, the Canadian investor on average suffers a 0.25 percent capital loss in USD terms, which is to be subtracted from the 1 percent gain on the USD itself.

- Lastly, consider a US importer. An appreciation of the USD relative to the CAD is good news for this US firm, because its costs have gone down. Thus, for a US importer, we would typically see a rise of the USD stock price, reinforcing the effect that the exchange rate itself has on the asset's CAD value. Thus, the gamma would exceed unity.

We conclude that exchange rate covariance risks can be very different for different assets. The relative exposure of a foreign T-bill is unity, but the relative exposure of a foreign stock could be higher, or lower. Notably, there is a whole group of foreign firms with gamma's below 1, and a bunch of domestic firms with gamma's above 0. We'd probably better speak of all of these as internationally competing firms that do not fundamentally differ from each other. In short, unlike T-bills, their stocks have no clear-cut economic nationality.

19.3.4 The International CAPM

Let us again consider the two equations that determine the Canadian and US market portfolios:

$$\text{CDN: } \mathrm{E}(\tilde{r}_j - r) = \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_p), \tag{19.31}$$
$$\text{US: } \mathrm{E}(\tilde{r}_j - r) = \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1-\lambda) \operatorname{cov}(\tilde{r}_j, \tilde{s}). \tag{19.32}$$

In Technical Note 19.3 it is shown that these equations can be aggregated into the following:

$$\mathrm{E}(\tilde{r}_j - r) = \lambda \operatorname{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \operatorname{cov}(\tilde{r}_j, \tilde{s}), \tag{19.33}$$

with $\tilde{r}_w$ referrring to the return on the world market portfolio and κ being a function of the national invested wealths and the national (unity minus) risk aversions. Compared to the country-by-country efficiency conditions, what we now have on the right-hand side is a covariance with the *world* market portfolio, which is more observable than the national portfolios, and a covariance with exchange rate, the result of taking into account the heterogeneous expectations induced by exchange rate uncertainty.

This is, again, half a CAPM in the sense that it tells us what expected returns should be, taking into account the risks of the assets. As before, we need to know the prices of risk before this is of any use whatsoever to an investor or analyst. The approach is the same as before except that we now need two benchmarks. If we pick the world market portfolio and the USD treasury bill, a simple generalization of the one-country CAPM emerges, as shown in Technical Note 19.4:

$$\mathrm{E}(\tilde{r}_j - r) = \beta_{j,w}\mathrm{E}(\tilde{r}_w - r) + \gamma_{j,s}\mathrm{E}(\tilde{s} + r^* - r), \tag{19.34}$$

where beta and gamma are from the multiple regression that combines the market model and the exposure model we considered in the preceding subsection:

$$\tilde{r}_j = \alpha_{j,w,s} + \beta_{j,w;s}\tilde{r}_w + \gamma_{j,s;w}\tilde{s} + \tilde{\epsilon}_{j;w,s}. \tag{19.35}$$

The subscript $j;s$ to beta intends to remind you that this is not the simple beta we are used to: we are now holding constant the exchange rate. Likewise, the subscript $j;w$ to gamma tells you we are now holding constant the world market return, unlike in the simple exposure regression we looked at a few pages up.

To interpret the regression [19.35] and the International CAPM [19.34], note that the regression again identifies the best possible replication of asset j that one can achieve using the two benchmark portfolios, the world market portfolio and the foreign T-bill, along with the risk-free asset.

Example 19.11

Suppose that, for a US stock, the coefficients in Equation [19.35] are estimated as $\beta_{j,w;s} = 1.2$ and $\gamma_{j,s;w} = 0.75$. Consider portfolios that consist of an investment in the world market portfolio (with weight x_w), an investment in the USD T-bill (with weight x_s), and weight $1 - x_w - x_s$ invested in the CAD risk-free asset. If $\beta_j = 1.2$ and $\gamma_j = 0.75$, we invest $x_w = 1.2$ in the world market portfolio, $x_s = 0.75$ in the USD T-bill, and 1 – 1.20 – 0.75 = –0.95 in the domestic risk-free asset. This portfolio provides the best possible replication of the return from asset j using just the two benchmark portfolios as replicating instruments.

The International CAPM then says that the expected return on a stock j is the same as the expected return on the stock's best replication portfolio—see Technical Note 19.5 for the details:

Example 19.12

Continue the same example ($\beta_{j,w;s} = 1.2$ and $\gamma_{j,s;w} = 0.75$). If the world market portfolio has an estimated risk premium of 0.05 and the currency of 0.01 *p.a.*, then the expected risk premium on the stock is estimated as $1.2 \times 0.05 + 0.75 \times 0.01 = 0.0675$, or 6.75 percent (on top of the risk-free rate).

19.3.5 The N-Country CAPM

The "world" (in the sense of the integrated capital market) has far more countries than two. The generalisation of the two-country model is obvious. First, there will be as many gamma's as there are exchange rates in the world. Second, the beta and the gammas must be estimated from one regression containing r_w and all the $\tilde{s}_i$'s:

$$E(\tilde{r}_j - r) = \beta_{j,w;..}E(\tilde{r}_w - r) + \gamma_{j,s_1;..}E(\tilde{s_1} + r_1^* - r) + \gamma_{j,s_2;..}E(\tilde{s_2} + r_2^* - r) + ...\gamma_{j,s_n;..}E(\tilde{s_n} + r_n^* - r), \quad (19.36)$$

where beta and the n gammas are from the multiple regression that combines the market model and n exposure models, one per currency, that we considered in the preceding subsection:[12]

$$r_j = \alpha_{j,w;\underline{s}} + \beta_{j,w;..}r_w + \gamma_{j,s_1;..}\tilde{s_1} + \gamma_{j,s_2;..}\tilde{s_2} + ...\gamma_{j,s_n;..}\tilde{s_n} + \epsilon_{j;w,\underline{s}}. \quad (19.37)$$

In practical applications, restraint is recommendable, as Goethe would readily concur. A CAPM *cum* regression of 150 terms will not do: it would add more noise than information. One reason is that exchange-risk premia $E(\tilde{s} + r^* - r)$ are empirically small, have a long-run mean that is hard to statistically distinguish from zero, and are not easy to estimate with reasonable precision. Also, gammas are similarly difficult to estimate precisely. So my advice is to surely restrict, *a priori*, the list of countries to those where there is a good common-sense reason for expecting an exposure, and censor away the gammas with the wrong size or

[12] Apologies for the baroque subscripts. The semi-colon usually initiates a list of variables that are held constant. Here the list would be too long, so we drop it. Still, you should remember that these are multiple-regression coefficients, measuring the impact of one variable holding constant the other ones.

sign. Personally I would perhaps even entirely omit the exposure terms: given the uncertainties surrounding the risk premia and the exposures, one might just work with the world-market term in the i-CAPM, and simply widen the scope of the sensitivity analysis that should be part and parcel of every capital-budgeting exercise:

$$\mathrm{E}(\tilde{r}_j - r) \approx \beta_{j,w;..}\mathrm{E}(\tilde{r}_w - r). \tag{19.38}$$

The only surviving difference with the standard CAPM would then be the use of a world market as benchmark, and the multiple beta.[13]

19.3.6 Empirical Tests of the International CAPM

In this chapter, we are suggesting that you replace your familiar single-market CAPM equation by a more complicated version, Equation [19.36] or [19.38]. The first issue is whether one of the basic assumptions of the international model, the absence of controls on capital flows, is reasonable. Second, are the empirical data compatible with the International CAPM and, if so, can we also reject the single-country view of the world?

Let us first examine the effect of direct controls on foreign investment. The controls may either limit foreign investment into a country or restrict domestic residents from investing abroad. Restrictions on foreign investment into a country may be imposed in different ways—in the form of a limit on the fraction of equity that can be held by foreigners or a restriction on the types of industries in which foreigners can invest. Historical details on the type and magnitude of these restrictions can be found in Eun and Janakiramanan (1986, Table 1). There may also be domestic controls on how much a resident can invest abroad. For example, Japanese insurance companies could not invest more than 30 percent of their portfolio in foreign assets at the time, and only 30 percent of Spanish pension funds could be invested abroad. Two questions need to be answered. One, if these restrictions exist, do they have a significant impact on the choice of the optimal portfolio and hence, potentially, on asset pricing? Two, how significant are these constraints today?

Bonser-Neal, Brauer, Neal and Wheatley (1990) examine whether the restrictions on investing abroad are binding. They look at closed-end country funds and find that these mutual funds trade at premia relative to their net asset values, indicating

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[13]The need to still use a multivariate regression even in the truncated model follows from the fact that our basic model is Equation [19.33], not Equation [19.23]. Equation [19.33] simplifies to the univariate equation, [19.23], only if either the prices of exchange covariance risk, η_k, are all zero, or the covariances between asset returns and exchange rate changes themselves are zero. The first case requires very special utility functions (with $\lambda = 1$), and the second case cannot possibly be true for all assets and home currencies simultaneously. Thus, we do need the multivariate model. Moreover, although the risk premium for exchange risk can be small it is unlikely to be exactly zero. That is, we use the one-factor world model merely as an approximation. If we would, in addition, use a univariate beta, we would introduce another (unnecessary) error to the approximation.

that the French, Japanese, Korean, and Mexican markets are at least partially segmented from the US capital market. Hietala (1989) studies the effects of the Finnish law that prevented investors from investing in foreign securities and finds that there is a significant difference between the returns on domestic assets required by residents compared to foreigners. Gultekin, Gultekin, and Penati (1989) find strong evidence that the US and Japanese markets were segmented prior to 1980. However, while there were substantial controls on capital flows before the 1980s, this is no longer true. Halliday (1989) already reports that even in those days there were few constraints on investing in foreign stock markets. This was and is especially true for investing in the markets of developed countries. For example, already in the 1980s there were no controls on international investment into or from Austria, Belgium, Denmark, Ireland, Italy, Japan, the Netherlands, the UK, the US, and West Germany. The controls studied by Hietala (1989) and Gultekin, Gultekin, and Penati (1989) were removed in 1986 and 1980, respectively. Also, looking at restrictions that limit domestic residents from investing abroad, one sees that these constraints are often not binding. For example, Fairlamb (1989) reports that in 1988 only 8 percent of Spanish funds were actually invested in foreign assets, while the limit was 30 percent. Thus, while direct controls on foreign investment may have been important in the past, they are probably no longer an important determinant of portfolio choice and asset pricing in the main OECD countries.

Let us now discuss the more direct tests of international asset pricing models. Solnik (1973), who did the first theoretical and empirical work in international asset pricing, tests a special case of Equation [19.36], where the world market risk premium and exposure risk premia could be merged into one single term. He concludes that the data are consistent with his International CAPM, although he does not test his model against the single-country alternative.

An early test that does compare an international asset pricing model against the single-country alternative was carried out by Stehle (1976). Specifically, Stehle tries to find out empirically whether US stocks are priced in a national market or in a world market. He, too, uses a restricted version of Equation [19.36], assuming that λ equals unity so that all currency risk premia disappear. The only remaining difference between the international model (Equation [19.36]) and the national model is the definition of the market portfolio. Specifically, in Equation [19.36], the benchmark portfolio is the world market portfolio, while in Equation [19.24], it is the national market portfolio. Stehle's tests are not able to empirically reject one in favor of the other, and Stehle concludes that asset pricing is done in a single-market context. Dumas (1976), however, argues that when the data do not allow one to distinguish between single-country asset pricing and international asset pricing, then one ought to retain the simplest view—that is, one should conclude that there is one international market instead of the many separate national markets.

There have been many additional empirical investigations, with a large portion of them testing special restricted versions of Equation [19.36]. The conclusions tended to be ambiguous. But more recent work has come up with more definite answers. As already mentioned, Gultekin, Gultekin, and Penati (1989) provide strong evidence that the US and Japanese markets were segmented prior to 1980. However, they also show that after the enactment of the Foreign Exchange and Foreign Trade Control law in 1980, there is no longer any significant evidence against the hypothesis that US and Japanese stocks are priced in an integrated market. A careful, and more recent, test is by Dumas and Solnik (1991). They test the Solnik-Sercu International CAPM, allowing for changes in risks and risk premia over time. Using data from major OECD countries, they reject Stehle's hypothesis that the exposure risk premia, γ_i, are zero, but they do not reject the full version (with non-zero risk premia for exchange rate exposure). They also reject single-country asset pricing (with a purely local benchmark). All of this lends support to the International CAPM, at least for the major OECD countries that do not impose explicit restrictions on capital movement. There are also a few papers by De Santis and Gerard (1997, 1998) that allow for autocorrelation in not just expected returns but also in variances and covariances, modeling the fact that risk comes in waves. Their work confirms that exchange-rate exposure is often non-zero and earns a statistically significant premium.

19.4 The CFO's Summary *re* Capital Budgeting

International asset pricing is potentially complicated by two extra issues: exchange risk, and segmentation of capital markets. If the capital market of the home country and the host country are integrated, the cash flows of an investment project can be valued in any currency, including the host currency. This simplifies capital budgeting in the sense that no exchange rate forecasts seem to be necessary for the translation. On the other hand, in integrated markets it becomes impossible to observe the portfolio of risky assets held by the average investor in any of the individual countries. Thus, the International CAPM has to be used, which means that, in principle, exchange rate expectations and exposures still show up in the cost of capital. In short, forecasts and exposures can only be eliminated by cutting corners.

Thus, the first issue is whether or not there is integration. Having selected either the single-country CAPM or the International CAPM, the next issue is to obtain estimates of the model parameters. We need the stock market sensitivity or beta and, in the International CAPM, the exchange rate exposures. We also need the expected return on the corresponding benchmark portfolios.

19.4.1 Determining the Relevant Model

If the capital market of the home country and the host country are segmented from each other, the investing firm should set the cost of capital equal to the return that is expected by its own shareholders. This means that a particular investment may be profitable for a foreign firm but not profitable for a local firm.

Table 19.3: **Rules for the Capital-Budgeting Process: Overview**

	CoCa model	currency of calculations
1. Foreign investments:		
• home and host financially integrated	inCAPM	FC & HC
• home and host financially segmented		
• home country part of larger financial market	inCAPM	HC only
• home country totally isolated	CAPM	HC only
2. domestic investments		
• home country part of larger financial market	inCAPM	n.a.
• home country totally isolated	CAPM	n.a.

Example 19.13

At the time of writing, the Chilean stock market remains strongly segmented from the rest of the world. If a Chilean firm makes an investment in Chile, the firm will estimate the beta by regressing returns from a portfolio of stocks in the same industry on the Chilean stock market index. Note that the returns from this investment are likely to be strongly correlated with the Chilean market index because there are important common factors, like the business cycle or interest rates, that affect all Chilean firms in similar ways. Thus, the investment is relatively risky for an Chilean firm. But the same project may be low-risk from the point of view of, say, an Austrian firm. The reason is that, because the Chilean economy is only loosely connected to North-America, Europe and Asia, the returns from the Chilean project will not be highly correlated with the returns on the typical world investor's portfolio, which is strongly diversified internationally. So the investment adds little to the risk of an international portfolio, but much more to the risk of a purely Chilean portfolio.

Note that segmentation of the home-country and the host-country capital markets does not mean that each market is a single-country market. The shareholders of the Austrian firm are likely to live in many different countries, and they all have access to non-Austrian shares, too. Thus, it is appropriate for the Austrian firm to set its cost of capital using an international model, that is, using the "world" market portfolio as a proxy for the true benchmark relevant to its shareholders.

19.4.2 Estimating the Risk of a Project

The market risk and the exchange risk exposures are defined as the slope coefficients in the regression of j's return on the world market return and all relevant exchange rate changes. Estimates obtained from time series of past data are subject to substantial estimation errors, stemming from pure sample-specific coincidences.

A standard solution is to estimate the risks from returns on industry portfolios rather than from individual stock data. That is, one estimates returns on, typically, an equally weighted portfolio of all stocks in the same industry i: One then estimates the risks by regressing industry-portfolio returns rather than individual stock returns. The underlying idea is that, as portfolio returns are more diversified, there is less residual noise in the regression, which improves the quality of the estimates.

Example 19.14

Suppose that Toyota considers building a new plant in the UK, which would sell its output in the entire European Union. Then Toyota could estimate the beta and gammas of the European car industry as a whole, rather than estimating the risks using just a simple stock.

Still, the portfolio approach assumes that all firms in the index have the same risks. In practice, one would often have serious difficulties in identifying a sufficiently large number of firms that have the same exposure as the project at hand.

Example 19.15

Suppose that Oerlikon, a Swiss firm, wants to build a plant for the production and sale of maintenance welding electrodes in India. There may be a number of Indian firms active in the welding industry, but not one of them is priced in the OECD capital market. Hence, Oerlikon cannot directly measure the risk of the Indian welding industry relative to the world market portfolio.[14] Thus, when valuing the project, Oerlikon would have to use an indirect, forward-looking approach to assess the risk. For instance, Oerlikon could argue that (1) the maintenance welding industry is not very cyclical, (2) the Indian business cycle is still largely independent of economic cycles in the OECD, so that (3) the beta of this Indian project relative to the OECD market portfolio is bound to be low. In addition, Oerlikon could argue that the exposures of Rupee cash flows to OECD exchange rates are small or zero because the Indian economy is still relatively closed. In short, beta is probably low; the Rupee gamma is probably equal to unity or thereabouts (as cashflows are unexposed in Rupee terms); and the other gammas must be close to zero.

Data availability is just one possible issue. The relevance of any available data is another. As pointed out in Chapter 13, exchange risk exposure when you are at the top of a PPP-deviation cycle would be very different from an exposure when the currency is at a low, in real terms. In such case, rather than estimating a misleading gamma you could (*i*) work with forward-looking scenarios, see Chapter 13 and then hedge the currency effect on the basis of the implied exposure; or (*ii*) ignore currency elements in the cost of capital, and widen the range of the sensitivity analyses.

[14] A procedure that consists of translating rupee returns on Indian stocks into an OECD currency and then estimating the risks is flawed because the prices of these Indian companies in the Bombay stock market are different from what they would have been if the assets had been priced internationally.

19.4.3 Estimating the Risk premia

Assuming that we have an approximate idea of the beta and gammas, we need estimates of the expected risk premia per unit of risk. The expected excess return on the world market portfolio is still rather hard to estimate, even though it is not quite as bad as a typical currency-risk premium. The sample averages of returns observed in the past differ substantially across sample periods, and it is also known that the expected return changes over time.[15] Still, we know that there is a positive risk premium on the world stock market, and variations over time in the expected excess return may not be overly important when the NPV evaluation horizon is, say, one decade rather than a month or two days.

Turning to the expected excess return on the various foreign T-bills, these risk premia also change over time, as we have seen in Chapter 10—and, unlike for the world market risk premium, we are not even sure whether the long-run mean actually differs from zero. Since exchange risk premia are small in the short run and close to zero in the long run, for practical applications one might have to be content with an approach that ignores these and use the following simplified version of Equation [19.36]:

$$\mathrm{E}(\tilde{r}_j - r) \approx \beta_{j,w;\underline{s}} \mathrm{E}(\tilde{r}_w - r), \tag{19.39}$$

where the beta is still estimated from a multivariate regression (Equation [19.37]) rather than from a bivariate regression).

You should not be overly discouraged by these approximations. No model is perfect; and the International CAPM does work better than competing models. Still, the cost of capital is measured imperfectly, and NPV computations should always be undertaken for a whole range of reasonable discount rates, to see to what extent the accept/reject recommendation is sensitive to the estimate of the cost of capital.

[15]The return is partially predictable on the basis of (1) the risk spread (the difference between low-grade bond yields and government bond yields), (2) the term spread (the difference between short-term and long-term bond yields), and (3) the dividend yield.

19.5 Technical Notes

Technical Note 19.1 The efficiency condition

Let the desirability of the portfolio p be denoted by $V_p = V(\mathrm{E}(\tilde{r}_p - r), \mathrm{var}(\tilde{r}_p))$. The optimum is found by setting, for each risky asset j, the derivative of V_p w.r.t. x_j equal to zero. The effect of a small change in x_j on V_p works through two channels: the expectation, and the variance; so below we see x_j's effect on V_p via the mean, and similarly x_j's effect on V_p via the variance. In the second line we fill in the effect of x_j on mean and variance, Equations [19.16] and [19.17]:

$$\begin{aligned} 0 &= \frac{\partial V}{\partial x_j} = \frac{\partial V}{\partial \mathrm{E}()}\frac{\partial \mathrm{E}()}{\partial x_j} + \frac{\partial V}{\partial \mathrm{var}()}\frac{\partial \mathrm{var}()}{\partial x_j}, \\ &= \frac{\partial V}{\partial \mathrm{E}()}\mathrm{E}(\tilde{r}_j - r) + \frac{\partial V}{\partial \mathrm{var}()}2\mathrm{cov}(\tilde{r}_j, \tilde{r}_p); \\ \Rightarrow 0 &= \mathrm{E}(\tilde{r}_j - r) - \lambda_p \mathrm{cov}(\tilde{r}_j, \tilde{r}_p), \end{aligned} \tag{19.40}$$

where $\lambda_p := -2\frac{\partial V/\partial \mathrm{var}()}{\partial V/\partial \mathrm{E}()}$. This is a positive number since a higher variance lowers the desirability V while a higher expected return increases it. Crypto-mathematicians recognize this ratio of partial derivatives as the implicit derivative (or marginal trade-off) of mean for variance in the chosen solution.

Technical Note 19.2 Using Ito's Lemma to transcribe the fc efficiency condition.

Start by relating the CAD return on j to the USD return: $1+\tilde r_j = (1+\tilde r_j^*)(1+\tilde s)$, with $\tilde s = \Delta S/S$ and S is CAD/USD. Solve for $\tilde r_j^*$ and Taylor-expand as follows:

$$\tilde r_j^* = \frac{1+\tilde r_j}{1+\tilde s} - 1 \approx \tilde r_j - \tilde s - [\tilde r_j\,\tilde s] + \tilde s^2, \tag{19.41}$$

A readily acceptable result of Ito's Lemma is that, for shorter and shorter holding periods, products of three or more returns become too small to matter. This, firstly, justifies the above second-order expansion. It also means that if we consider covariances of two FC returns we only need to look at first-order terms:

$$\begin{aligned}\text{cov}(\tilde r_j^*,\,\tilde r_k^*) &\approx \text{cov}(\tilde r_j-\tilde s,\,\tilde r_k-\tilde s),\\ &= \text{cov}(\tilde r_j,\tilde r_k) - \text{cov}(\tilde r_j,\tilde s) - \text{cov}(\tilde r_k,\tilde s) + \text{var}(\tilde s).\end{aligned} \tag{19.42}$$

because all the other terms would lead to products of three or four returns.

One often reads that also inside an expectation only the first-order terms matter, because products of returns are second order of smalls. But this is patently wrong. Indeed, variances and covariances of returns are averages of products of two returns, but this surely does not mean that they can be set equal to zero. Now the expectation of, say, the third term is

$$\text{E}(\tilde r_j^*\,\tilde s) = \text{E}(\tilde r_j^*)\,\text{E}(\tilde s) + \text{cov}(\tilde r_j^*,\,\tilde s). \tag{19.43}$$

If we let the periods over which one observes return become shorter and shorter, all means and all (co)variances shrink roughly in proportion to the time interval Δt, so they preserve the same relative order of magnitude relative to each other. But this means that the product of two means, $\text{E}(\tilde r_j^*)\,\text{E}(\tilde s)$, shrinks to zero much faster than the covariance. That is, the product of two means is second order of smalls but the covariance is not:

$$\text{E}(\tilde r_j^*\,\tilde s) \approx \text{cov}(\tilde r_j,\,\tilde s)\text{ , and} \tag{19.44}$$

$$\text{E}(\tilde s^2) \approx \text{var}(\tilde s); \tag{19.45}$$

Using the above in Equation [19.41], we get the following translated expected return:[16]

$$\text{E}(\tilde r_j^*) \approx \text{E}(\tilde r_j) - \text{E}(\tilde s) - \text{cov}(\tilde r_{p^*}^*,\tilde s) + \text{var}(\tilde s). \tag{19.46}$$

Our results [19.46] and [19.42] for the translated mean and variance imply that the efficiency condition [19.27] translates into the first equation below. We next write that equation for the special case where asset j is the HC risk-free asset, and lastly we subtract:

$$\begin{array}{llllllll}
\text{E}(\tilde r_j)- & \text{E}(\tilde s) & -\text{cov}(\tilde r_j,\tilde s) & +\text{var}(\tilde s) & =\lambda\,[\text{cov}(\tilde r_j,\tilde r_{p^*}) & -\text{cov}(\tilde r_j,\tilde s) & -\text{cov}(\tilde r_{p^*},\tilde s)+var(\tilde s)]\\
r- & \text{E}(\tilde s) & -0 & +\text{var}(\tilde s) & =\lambda\,[0 & -0 & -\text{cov}(\tilde r_{p^*},\tilde s)+var(\tilde s)]\\ \hline
\text{E}(\tilde r_j)-r & & -\text{cov}(\tilde r_j,\tilde s) & & =\lambda\,[\text{cov}(\tilde r_j,\tilde r_{p^*}) & -\text{cov}(\tilde r_j,\tilde s)] & & ,
\end{array}$$

which leads to [19.32].

[16] Note, in passing, how we find back our earlier numerical result that covariance between CAD asset return and the CAD/USD exchange rate lowers the expected USD return. We also discover that exchange risk has its impact on the expected return too. So both the covariance and the variance have both 'good' and 'bad' aspects.

Technical Note 19.3 Aggregating the two efficiency conditions.

We want to aggregate, and obtain the world-market return, which is defined as

$$\tilde{r}_w = \frac{W_{ca}\tilde{r}_p + W_{us}\tilde{r}_{p^*}}{W_{ca} + W_{us}} \tag{19.47}$$

with W_{ca} and W_{us} defined as the invested wealths, both measured in CAD, of Canada and the US, respectively. To build this world return into the model we multiply both sides of [19.31] by W_{ca}, and [19.32] by W_{us}. On the right-hand sides of the equations below we have immediately put these factors inside the covariances. Next we sum the two equations, and lastly we divide by total world wealth and use [19.47]:

$$\begin{array}{rll}
W_{ca}\mathrm{E}(\tilde{r}_j - r) = & \lambda\,\mathrm{cov}(\tilde{r}_j,\, W_{ca}\,\tilde{r}_p) & \\
W_{us}\mathrm{E}(\tilde{r}_j - r) = & \lambda\,\mathrm{cov}(\tilde{r}_j,\, W_{us}\,\tilde{r}_{p^*}) & +W_{us}(1-\lambda)\,\mathrm{cov}(\tilde{r}_j, s) \\
\hline
(W_{ca} + W_{us})\mathrm{E}(\tilde{r}_j - r) = & \lambda\,\mathrm{cov}(\tilde{r}_j,\, (W_{ca}\,\tilde{r}_p + W_{us}\,\tilde{r}_{p^*})) & +W_{us}(1-\lambda)\,\mathrm{cov}(\tilde{r}_j, s) \\
 & & \\
\Rightarrow \mathrm{E}(\tilde{r}_j - r) = & \lambda\,\mathrm{cov}(\tilde{r}_j,\, \tilde{r}_w) & +\frac{W_{us}}{W_{ca}+W_{us}}(1-\lambda)\,\mathrm{cov}(\tilde{r}_j, s).
\end{array}$$

For ease of manipulation, in [19.33] we denote $W_{us}/(W_{ca} + W_{us})(1 - \lambda) = \kappa$.

Technical Note 19.4 Identifying λ and κ.

Write the equation in matrix form,

$$\mathrm{E}(\tilde{r}_j - r) = [\mathrm{cov}(\tilde{r}_j, r_w)\ ,\ \mathrm{cov}(\tilde{r}_j, \tilde{s})] \begin{bmatrix} \lambda \\ \kappa \end{bmatrix}. \tag{19.48}$$

To identify λ and κ we write this for two benchmarks, the world market portfolio with return r_w and the USD T-bill with return $r^* + \tilde{s}$;

$$\begin{bmatrix} \mathrm{E}(\tilde{r}_w - r) \\ r^* + \mathrm{E}(\tilde{s}) - r \end{bmatrix} = \begin{bmatrix} \mathrm{var}(\tilde{r}_w) & \mathrm{cov}(\tilde{r}_w, \tilde{s}) \\ \mathrm{cov}(\tilde{r}_w, \tilde{s}) & \mathrm{var}(\tilde{s}) \end{bmatrix} \times \begin{bmatrix} \lambda \\ \kappa \end{bmatrix}; \tag{19.49}$$

$$\Rightarrow \begin{bmatrix} \lambda \\ \kappa \end{bmatrix} = \begin{bmatrix} \mathrm{var}(\tilde{r}_w) & \mathrm{cov}(\tilde{r}_w, \tilde{s}) \\ \mathrm{cov}(\tilde{r}_w, \tilde{s}) & \mathrm{var}(\tilde{s}) \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{E}(\tilde{r}_w - r) \\ r^* + \mathrm{E}(\tilde{s}) - r \end{bmatrix}. \tag{19.50}$$

This can be substituted back into [19.48]. Now the covariance matrix of $(\tilde{r}_w,\ \tilde{s})$ premultiplied by the vector of covariances of r_j with these same variables $(\tilde{r}_w,\ \tilde{s})$ is the row vector of multiple regression coefficients of r_j onto $(\tilde{r}_w,\ \tilde{s})$—a generalisation of $b = \mathrm{cov}(\tilde{y}, \tilde{x}) \times \mathrm{var}(\tilde{x})^{-1}$ in $\tilde{y} = a + b\tilde{x} + \tilde{e}$:

$$\begin{aligned} \mathrm{E}(\tilde{r}_j - r) &= [\mathrm{cov}(\tilde{r}_j, r_w)\ ,\ \mathrm{cov}(\tilde{r}_j, \tilde{s})] \begin{bmatrix} \mathrm{var}(\tilde{r}_w) & \mathrm{cov}(\tilde{r}_w, \tilde{s}) \\ \mathrm{cov}(\tilde{r}_w, \tilde{s}) & \mathrm{var}(\tilde{s}) \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{E}(\tilde{r}_w - r) \\ r^* + \mathrm{E}(\tilde{s}) - r \end{bmatrix} \\ &= [\beta_{j,w;s}\ ,\ \gamma_{j,s;w}] \begin{bmatrix} \mathrm{E}(\tilde{r}_w - r) \\ r^* + \mathrm{E}(\tilde{s}) - r \end{bmatrix}. \end{aligned} \tag{19.51}$$

Technical Note 19.5 The best-replication reading of the i-CAPM

The claim can be shown as follows. In the first line, we write the return on a general portfolio with weights x_w and x_s for the world market and the foreign T-bill, and in the second line we group terms in x_w and x_s:

$$\begin{aligned} \mathrm{E}(\tilde{r}_{\text{j's replication}}) &= x_w \mathrm{E}(\tilde{r}_w) + x_s(r^* + \tilde{s}) + (1 - x_w - x_s)r && (19.52) \\ &= r + x_w \mathrm{E}(\tilde{r}_w - r) + x_s(r^* + \mathrm{E}(\tilde{s}) - r). && (19.53) \end{aligned}$$

For best replication, we have to set $x_w = \beta_{j,w;s}$ and $x_s = \gamma_{j,s;w}$. Thus,

$$\mathrm{E}(\tilde{r}_{\text{j's replication}} - r) = \beta_{j,w;s} \mathrm{E}(\tilde{r}_w - r) + \gamma_{j,s;w}(r^* + \mathrm{E}(\tilde{s}) - r). \quad (19.54)$$

19.6 Test Your Understanding: basics of the CAPM

19.6.1 Quiz Questions

True-False Questions

1. The risk of a portfolio is measured by the standard deviation of its return.
2. The risk of an asset is measured by the standard deviation of its return.
3. Each asset's contribution to the total risk of a portfolio is measured by the asset's contribution to the total return on the portfolio.
4. A risk-averse investor always prefers the highest possible return for a given level of risk or the lowest risk for a given level of expected return.
5. The means and standard deviations of all optimal portfolios selected from a risk-free asset and a set of risky assets are found on the line that originates at r_0 and is tangent to the efficient portfolio of risky assets.
6. Relative risk aversion shows the price in currency units of a given amount of risk.
7. Relative risk aversion varies from asset to asset because some assets are riskier than others.
8. Portfolio theory assumes that all investors are equally risk averse.

Multiple-Choice Questions

1. When using portfolio theory, we must make a number of assumptions. Which of the following assumptions are made? Which are not?
   (a) The rates of inflation at home and abroad are equal.
   (b) There are no information or transactions costs.
   (c) There are no taxes.
   (d) Investors want to know the distribution of wealth at the end of the period.
   (e) Investors care about the future expected return on their portfolio and the variability of this return.

19.6.2 Applications

1. The Country Prince Rupert's Land (PRL) has two companies, Hudson Bay Company (HBC) and Boston Tea Traders (BTT). In equilibrium, the returns of these two companies have the following distributions:

	Expected excess return	Covariances HBC	BTT
HBC	0.11	0.04	0.01
BTT	0.08	0.01	0.02

(a) Vary the weight of HBC from 0 to 1 by increments of 0.1, and compute how the portfolio covariance risks of HBC and BTT change as a function of the weights x_{HBC} and $x_{BTT} = 1 - x_{HBC}$.

(b) Find the optimal weights of x_{HBC} and $x_{BTT} = 1 - x_{HBC}$ and the average risk aversion.

(c) If the total value of the PRL stock market portfolio is 1,000, what is the value of HBC and BTT?

2. Consider the following covariance matrix and expected return vector for assets 1, 2, and 3:

$$V = \begin{bmatrix} 0.0100 & 0.0020 & 0.0010 \\ 0.0020 & 0.0025 & 0.0030 \\ 0.0010 & 0.0030 & 0.0100 \end{bmatrix} \quad E(\tilde{r}_j) = \begin{bmatrix} 0.0330 \\ 0.0195 \\ 0.0250 \end{bmatrix}$$

(a) Compute the expected return on a portfolio with weights for assets $j = 0, \ldots, 3$ equal to $[0.2, 0.4, 0.2, 0.2]$, when the T-bill (asset 0) yields a return of 1 percent. Do so directly, and then via the excess returns.

(b) Compute the variance of the same portfolio.

(c) Compute the covariance of the return on each asset with the total portfolio return, and verify that it is a weighted covariance.

(d) Is the above portfolio efficient?

(e) Are the following portfolios efficient?
- weights $(0.7, 0.1, 0.1, 0.1)$ for assets $j = 0, \ldots, 3$
- weights $(0.6, 0.2, 0.1, 0.1)$ for assets $j = 0, \ldots, 3$

(f) What is the portfolio held by an investor with risk-aversion measure $\lambda = 2.5$?

(g) Assume that there are no "outside" bills, that is, all risk-free lending and borrowing is among investors. Therefore the average investor holds only risky assets. What is the portfolio composition? What is the average investor's risk-aversion measure λ?

19.7 Test Your Understanding: iCAPM

19.7.1 Quiz Questions

True-False Questions

1. The entire NPV analysis can be conducted in terms of the host (foreign) currency if money markets and exchange markets are fully integrated with the home market.

2. The entire NPV analysis can be conducted in terms of the host currency if money markets, stock markets, and exchange markets are fully integrated with the home market.

3. Forward rates can be used as the risk-adjusted expected future spot rates to translate the host-currency cash flows into the home currency. The home-currency cash flows can then be discounted at the appropriate home-currency discount rate if money markets and exchange markets are fully integrated with the home market.

4. Regardless of the degree of market integration, the host-currency expected cash flows can always be translated into the home currency (by multiplying them by the expected spot rate), and then discounted at the home-currency discount rate.

5. Regardless of the degree of market integration, the host-currency expected cash flows can always be translated into expected cash flows expressed in home currency. The home-currency cash flows can then be discounted at the home-currency discount rate that takes into account all risks.

6. If you use the forward rate as the risk-adjusted expected spot rate, there is no need to worry about the dependence between the exchange rate and the host-currency cash flows.

7. If markets are integrated and you translate at the forward rate, the cost of capital need not include a risk premium for exchange rate exposure.

8. If markets are integrated and you translate at the forward rate, the cost of capital need not include a risk premium for exposure to any currency.

9. If you discount expected cash flows that are already expressed in home currency, the cost of capital should include a risk premium for exposure to the host-currency exchange rate.

10. If you discount expected cash flows that are already expressed in home currency, the cost of capital should include a risk premium for exposure to all relevant exchange rates.

11. If you translate at the forward rate, you can entirely omit exchange rate expectations from the NPV procedure.

12. Exchange rate risk premia are sizeable. In fact, they are about as large as the (world) market risk premium.

13. A highly risk-averse investor will only accept variance risk if he or she is fully certain to be compensated for this risk.

14. A highly risk-averse investor will never select a high-variance portfolio.

15. A risk-averse investor will select a high-variance portfolio only if the expected excess return is sufficiently high.

16. A risk-averse investor will select a low-return portfolio only if the variance is sufficiently low.

17. A particularly risk-averse investor will always select a low-return portfolio. This is because low return means low risk, and because the investor does not want to bear a lot of risk.

For the next set of questions, assume that access to money markets and exchange markets is unrestricted and the host-currency cash flow is risk free. Are the following statements true or false?

18. You can translate at the expected spot rate and discount at a risk-adjusted home-currency cost of capital.

19. You can translate at the forward rate, and discount at a home-currency rate that takes into account exchange risk.

20. You can translate at the forward rate, and discount at the risk-free home-currency rate.

21. You can discount the host-currency cash flows at the foreign risk-free rate, and then translate the result at the current spot exchange rate.

22. You can discount the host-currency cash flows at the foreign risk-free rate, and then translate the result at the expected future spot exchange rate.

23. You can discount the host-currency cash flows at the foreign risk-free rate, and then translate the result at the forward exchange rate.

24. If access to forward markets or foreign and domestic money markets is restricted, then the true value is always overstated if the foreign currency cash flow is translated at the forward exchange rate and then discounted at the domestic risk-free rate.

Additional Quiz Questions

1. Suppose that you observe an efficient portfolio. There are two methods with which you can infer the degree of risk aversion of the investor that selects this particular portfolio. What are these two methods?

2. What's wrong with the following statement: "The CAPM says that the expected return on a given stock j is equal to the best possible replication that one can obtain using the risk-free assets and the set of all risky assets (other than stock j)."

3. Below, we reproduce some equations from the derivation of the CAPM. Equation [20.1] is the efficiency criterion. Equation [19.62] is the CAPM. Explain the equations.

$$\frac{\mathrm{E}(\tilde{r}_j - r)}{\mathrm{cov}(\tilde{r}_j - \tilde{r}_m)} = \theta, \tag{19.55}$$

for all risky assets j=1,. . . ,N.

$$\begin{aligned}
\mathrm{E}(\tilde{r}_j - r) &= \theta\ \mathrm{cov}(\tilde{r}_j, \tilde{r}_m), && (19.56)\\
&= [\theta\ \mathrm{var}(\tilde{r}_m)]\frac{\mathrm{cov}(\tilde{r}_j, \tilde{r}_m)}{\mathrm{var}(\tilde{r}_m)}, && (19.57)\\
&= [\theta\ \mathrm{var}(\tilde{r}_m)]\beta_j, && (19.58)\\
\sum_{j=1}^{N} x_j \mathrm{E}(\tilde{r}_j - r) &= \theta\ , && (19.59)\\
\sum_{j=1}^{N} x_j \mathrm{cov}(\tilde{r}_j, \tilde{r}_m) &= \theta\ \mathrm{cov}(\sum_{j=1}^{N} x_j \tilde{r}_j, \tilde{r}_m), && (19.60)\\
&= \theta\ \mathrm{cov}(\tilde{r}_m, \tilde{r}_m), && (19.61)
\end{aligned}$$

$$\mathrm{E}(\tilde{r})_j - r = \beta_j[\mathrm{E}(\tilde{r}_m) - r]. \tag{19.62}$$

4. Suppose that investors from a country have access to a large set of foreign stocks, and that foreign investors can also buy stocks in that country. Which of the following statements is (are) correct?

   (a) The single-market CAPM, where the market portfolio is measured by the index of all stocks issued by local companies, does not hold.

   (b) The single-market CAPM, where the market portfolio is measured by the index of all stocks held by local investors, does not hold.

   (c) The single-market CAPM, where the market portfolio is measured by the index of all stocks held by local investors, is formally correct but not fit for practical use, because the correct index is not readily observable.

   (d) The single-market CAPM, where the market portfolio measured by the index of all stocks worldwide, is correct provided that there is a unified world market for all stocks.

(e) The single-market CAPM, where the market portfolio is measured by the index of all stocks worldwide, is correct provided that there is no (real) exchange risk.

19.7.2 Applications

1. Suppose that you have the following data:
Asset 0 is the (domestic) risk-free asset, and asset weights in a portfolio are denoted as x_j, where j = 0,.. ., 2. Which of the following portfolios is efficient, and if the portfolio is efficient, what is the investor's degree of risk aversion?

   (a) $x_0 = 0$, $x_1 = 0.4$, $x_2 = 0.6$
   (b) $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.4$
   (c) $x_0 = 0$, $x_1 = 0.5$, $x_2 = 0.5$
   (d) $x_0 = 0.2$, $x_1 = 0.4$, $x_2 = 0.4$
   (e) $x_0 = 0.5$, $x_1 = 0.25$, $x_2 = 0.25$
   (f) $x_0 = -1$, $x_1 = 1$, $x_2 = 1$
   (g) $x_0 = 1$, $x_1 = 0$, $x_2 = 0$
   (h) $x_0 = 2$, $x_1 = -0.5$, $x_2 = -0.5$

2. Suppose that the capital markets of the following three countries are well integrated: North America (with the dollar), Europe (with the EUR), and Japan (with the yen). Suppose that you choose the yen as the home currency.

   (a) Why does the average investor care about the JPY/USD and JPY/EUR exchange rates (beside how it relates to how his or her wealth is measured in JPY)?
   (b) What moments are needed in a mean-and-(co)variance framework, to summarize the joint distribution of asset returns? Which of these are affected by the portfolio choice?

3. Suppose that your assistant has run a market-model regression for a company that produces sophisticated drilling machines, and finds the following results (t-statistic in parentheses):

$$\tilde{r}_j = \alpha + \beta\tilde{r}_m + \gamma s + \tilde{e}_j,$$
$$\tilde{r}_j = \underset{(0.52)}{0.002} + \underset{(1.25)}{0.56}\tilde{r}_m + \underset{(2.06)}{4.25}\tilde{s} + \tilde{e}_j.$$

Your assistant remarks that, as the estimated beta is insignificant, the true beta is zero. The exposure, in contrast, is significant, and must be equal to the estimated coefficient. How do you react?

4. Suppose that the world beta for a German stock (in euro) equals 1.5, and its exposures to the dollar, the yen, and the pound are 0.3, 0.2, and 0.1, respectively.
   (a) What is the best replicating portfolio if you can invest in a world-market index fund, as well as in dollars, yens, pounds, and euros?
   (b) What additional information is needed to identify the cost of capital?

5. Suppose that there are two countries, the US (which is the foreign country) and Canada. The exposure of the company XUS, in terms of USD, is estimated as follows:

$$\tilde{r}^*_{XUS} = 0.12 + 0.30\tilde{s}_{USD/CAD} + \tilde{\varepsilon}.$$

   What is the company's exposure in terms of CAD?